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Physics. — *The spectrum of ionized Neon (Ne II).* First communication. By T. L. DE BRUIN. (Communicated by Prof. P. ZEEMAN.)

(Communicated at the meeting of May 28, 1927).

Introduction.

The spectrum, originating from the neutral atom, is called the arc spectrum. The energy levels or terms are of the form :

$$\underset{(arc)}{Term} = \frac{R}{(n+\triangle)^2}$$

where R = the RYDBERG constant (= 109737) n = the principal quantum number and $\triangle$ the so called quantum defect caused by the core of the atom. The spark spectrum has its origin in the ionized atom, the atom that has lost one electron. Now the energy levels have the form :

$$\underset{(ion)}{Term} = \frac{4R}{(n+\triangle)^2}$$

The arc spectrum of Neon (indicated by the symbol *Ne I*) is of great complexity, but by the fundamental work of PASCHEN [1]) now it is one of the best known spectra. MERTON [2]) has found, first, that the *Neon* atom under condensed discharges can emit another spectrum than the arc spectrum. This spectrum lies mainly in the blue-violet region. Careful new measurements of this spectrum have been made by L. BLOCH, E. BLOCH and G. DÉJARDIN [3]). This material we have used for the analysis of the *Ne II* spectrum. The analysis of the *F I* spectrum, given in former papers [4]) and the theory of complex spectra developed by RUSSELL, SAUNDERS, PAULI and specially by HEISENBERG and HUND, could be preliminary steps for this analysis.

Analogy between the Ne II atom and the F I atom.

In former papers in these Proceedings, I have given an analysis of the spectrum emitted by the neutral Fluorine atom (*F I*). The analysis has been briefly discussed in relation in the HEISENBERG—HUND theory of complex spectra, and a correlation with the scheme of terms predicted by the theory has been established. In that paper we noted already that the spectra of the ionized rare gases should have the same structure as the neutral halogen atoms. The *Ne II* spectrum should have the same structure as the *F I* spectrum. These atoms have the same configurations two 1_1, two 2_1 and five 2_2 electrons. In the case of *Fluorine I* the charge of the core is + 9 in the case of *Neon II* + 10.

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[1]) PASCHEN, Ann. d. Phys., **60**, 405, 1919; **63**, 201, 1920.

[2]) MERTON, Proc. R. S. London, *A*, **89**, 447, 1914.

[3]) L. BLOCH, E. BLOCH, G. DÉJARDIN, Journ. d. Phys., **7**, No. 5. 129, 1926.

[4]) T. L. DE BRUIN, Versl. Kon. Akad. Amsterdam, **35**, Juni 1926, Dec. 1926; Zeitsch. f. Phys., **39**, 869, 1926.

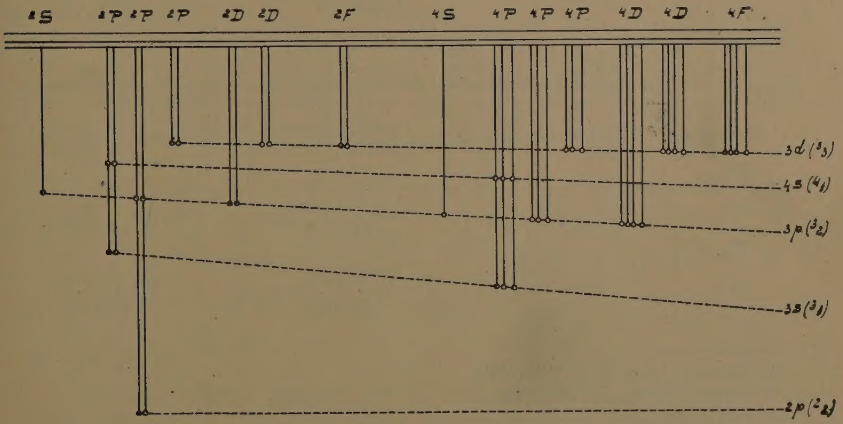
TABLE 1.

n_k	1_1	2_1	2_2	Nucleus
F I	2	2	5	+ 9
Ne II	2	2	5	+ 10

The theoretical termscheme for *Ne II*, deduced from the atom configuration, is quite the same as that for *F I* and the following terms are predicted by theory :

TABLE 2. Neon II (Deep terms).

Electron configuration							Basic term: ^3P	
							Terms	
n_k 1_1	2_1	2_2	3_1	3_2	3_3	4_1	Quartet	Doublet
2	2	5						*P*
2	2	4	1				*P*	*P*
2	2	4		1			*D P S*	*D P S*
2	2	4			1		*F D P*	*F D P*
2	2	4				1	*P*	*P*



Term scheme (*Ne II.*).

The ground term should be, as in the case of *F I*, a ^2P term with the term difference of the Röntgen L-doublet. This doublet we know for *Ne II* from the two series limits of the arc spectrum, which have the difference of 780 frequentie units. According to the formula of Sommerfeld for the Röntgen L-doublet:

$$\triangle\nu_L = 0.365\,(Z-s)^4 \qquad \begin{array}{l} Z = \text{atomic number} \\ s = \text{screening constant} \end{array}$$

we have $\triangle\nu = 780$, $Z = 10$, $s = 3.20$. In the case of *F I* for $Z = 9$ and $s = 3.20$ we find the value $\triangle\nu = 412$. This value agrees satisfactory with the experimental value (407) detected from the spectrum 1). The order of the ground doublet ^2P for *Ne II* will be: 33.4 Volt 2) $= \pm$ 270.000, for *F I* approximately $\pm$ 16.7 Volt 3) $= \pm$ 135.000.

The key for the analysis of the *F I* spectrum is given by the ^4P term with the term differences 160.0 and 274.6. For the analysis of the *Ne II* spectrum it was important to find the analogous term. In the case of *Ne II* this term has the differences 299.0 and 518.0. From this we can deduce the structure of the spectrum. The analoguous multiplets $^4(PP')$ are: (This multiplet in *F I* is found by me on basic of the Zeeman effects.)

TABLE 3.

Ne II	$^4P_1$	299.0	$^4P_2$	518.0	$^4P_3$
$^4P'_1$	(4) 3751.25 26650.2		(7) 3709.66 (*R*) 4) 26949.0		—
182.5					
$^4P'_2$	(8) 3777.14 (*R?*) 26467.5		(7) 3734.94 26766.6		(9) 3664.05 (*R*) 27284.4
222.7					
$^4P'_3$	—		(8) 3766.28 (*R?*) 26543.8		(10) 3694.19 (*R*) 27061.8
F I	$^4P_1$	159.9	$^4P_2$	274.6	$^4P_3$
$^4P'_1$	(0) 7515.0 13303.0		(12) 7426.2 13462.1		—
102.1					
$^4P'_2$	(5) 7573.32 13200.6		(5) 7482.95 13360.0		(18) 7332.25 13634.5
122.7					
$^4P'_3$	—		(5) 7552.20 13237.5		(20) 7398.96 13511.7

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1) Bowen: Phys. Rev., Febr. 1927.

2) Mohler: Science, April 1926.

3) Deduced from the termscheme.

4) *R* = reversed.

While this multiplet $^4(PP')$ in the case of *F I* lies far in the red, in the case of *Ne II* it lies far in the violet, as could be expected from the factor $4R$ in the expression for the term. The ratio for the ionisation potentials of *F I* and *Ne II* is approximately the same as for the frequentie numbers of the analogous multiplets above mentioned viz. 2.

Terms in the Ne II spectrum.

From the value of the groundterm ^2P in *Ne II* ± 33.4 Volt = ± 270000 we can deduce the order of the terms due to the binding in a 2_2 orbit to the ion. We have:

$$270.000 = \frac{4R}{n^2} \quad n = \text{effective quantum number}$$

with $n = 1.275$. The doublet terms arising from the binding of a 3_2 electron have approximately the value: $\frac{4R}{2.275^2} = \pm 82000$. The quartet terms arising from this binding lie somewhat deeper. (See figure.) The terms arising from the binding of a 3_3 electron have approximately the value: $\frac{4R}{3^2} = \pm 50.000$. Except the ground doublet we have also four groups of terms arising respectively from the binding of a 3_1, 3_2, 3_3 (4_1) electron and an electron with looser coupling.

In the *Ne II* spectrum I have found these four groups of terms. The deepest quartet terms are identified. The other terms are indicated with preliminary symbols. In another communication in these Proceedings we will give more details.

TABLE 4. Termtable *Ne II.*

Term-symbol	j	Term values (relative)	Term difference	Atom-configuration	Remarks
^4P	3	117000.0		$(s^2p^4).3_1$ [1]	
			518.0		
	2	116482.0			
			299.0		
	1	116183.0			

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[1] The symbol $(s^2p^4).3_1$ indicates that the atom configuration consists of the ion, having two *s*-electrons ($k = 1$) and four *p*-electrons ($k = 2$), and the binding of an 3_1 electron.

TABLE 4. Termtable *Ne II (Continued).*

Term-symbol	j	Term values (relative)	Term difference	Atom-configuration	Remarks
$^4P'$	3	89938.2	222.6	$(s^2p^4).3_2$	
	2	89715.6	182.5		
	1	89533.1			
^4D	4	87022.7	337.8	$(s^2p^4).3_2$	
	3	86684.9	249.7		
	2	86435.2	144.1		
	1	86291.1			
^4S	2	83178.0		$(s^2p^4).3_2$	
T	2	81965.7			
$^4D'$	(4)	56994.0	81.8	$(s^2p^4).3_3$	
	3	56912.2	106.0		
	2	56806.2	98.3		
	1	56707.9			
P_2	2(3)	55862.0			
xP	2	55657.4			
yP_3	3(1)	55333.7			
aP_2	2	55183.8			
bP_2	2	55141.4			
cP_3	3	55104.9			
dP_2	2	54959.4			
U	2	54411.5			
eP_3	3	54133.0			
fP_2	2	53755.6			
gP_1	1	53450.8			
nP_3	3	33287.2			
kP	(2,3)	33227.4			
mP_2	2	33141.4			
R_1	1	32621.4			
lP	(3)	32530.7			
oP_3	3	32306.2			
pP_3	3	23359.2			
qP_2	2	23327.5			
rP_1	1	23261.1			

TABLE 5.

	$^4P_{123} - {}^4P'_{123}$			
8	3777.14	26467.5	$^4P_1 - {}^4P'_2$	*R?*
8	3766.28	26543.8	$^4P_2 - {}^4P'_3$	*R?*
4	3751.25	26650.2	$^4P_1 - {}^4P'_1$	
7	3734.94	26766.6	$^4P_2 - {}^4P'_2$	
7	3709.66	26949.0	$^4P_2 - {}^4P'_1$	*R*
10	3694.19	27061.8	$^4P_3 - {}^4P'_3$	*R*
9	3664.05	27284.4	$^4P_3 - {}^4P'_2$	*R*
	$^4P_{123} - {}^4D_{1234}$			
2	3270.79	30564.9	$^4P_3 - {}^4D_2$	
5	3360.63	29747.8	$^4P_1 - {}^4D_2$	
7	3355.09	29796.9	$^4P_2 - {}^4D_3$	
5	3344.44	29941.8	$^4P_1 - {}^4D_1$	
10	3334.89	29977.3	$^4P_3 - {}^4D_4$	
5	3327.22	30046.5	$^4P_2 - {}^4D_2$	
4	3311.32	30190.8	$^4P_2 - {}^4D_1$	
8	3297.74	30315.1	$^4P_3 - {}^4D_3$	
	$^4P_{123} - S^4_2$			
4	3028.90	33005.7	$^4P_1 - {}^4S_2$	$^4P' - {}^4D'_1$
7	3001.72	33304.5	$^4P_2 - {}^4S_2$	
7	2955.77	33821.8	$^4P_3 - {}^4S_2$	
	$^4P'_{123} - {}^4D'_{123}$			
5 d.v.	3054.70	32726.9	$^4P'_1 - {}^4D'_2$	
6	3047.60	32803.2	$^4P'_2 - {}^4D'_3$	
3	3045.56	32825.1	$^4P'_1 - {}^4D'_1$	
3	3037.75	32909.5	$^4P'_2 - {}^4D'_2$	
5	3034.49	32944.9	$^4P'_3 - {}^4D'_4$	
4	3028.90	33005.7	$^4P'_2 - {}^4D'_1$	$^4P_1 - {}^4S_2$
4	3027.07	33025.6	$^4P'_2 - {}^4D'_1$	
3	3017.36	33131.9	$^4P'_3 - {}^4D'_2$	

TABLE 5 *(Continued).*

	$^4D_{1234} - {}^4D'_{1234}$			
1	3390.56	29485.2	$^4D_1 - {}^4D'_2$	
1	3386.20	29523.2	$^4D_2 - {}^4D'_3$	
0	3379.34	29582.7	$^4D_1 - {}^4D'_1$	
2	3374.08	29629.2	$^4D_2 - {}^4D'_2$	
6	3367.25	29689.3	$^4D_3 - {}^4D'_4$	
1	3362.90	29727.7	$^4D_2 - {}^4D'_1$	
2	3357.89	29772.1	$^4D_3 - {}^4D'_3$	$^4S_2 - gP_1$
1	3345.88	29878.9	$^4D_3 - {}^4D'_2$	
4	3329.18	30028.8	$^4D_4 - {}^4D'_4$	
1	3320.28	30109.3	$^4D_4 - {}^4D'_3$	
1	2895.05	34531.8	$^4P'_2 -$	*(Ne III ?)*
2	2910.44	34349.0	$^4P'_1 -$	
3	3173.58	31501.1	$^4D_3 -$ aP_2	
3	3213.77	31107.1	$^4D_1 -$	
3	3571.24	27993.5	$^4S_2 -$	
2	2872.96	34796.1	$^4P'_3 -$	
0	2891.50	34574.0	$^4P'_2 -$	$^4P' - dP_2$
1	2906.85	34391.4	$^4P'_1 -$	
0	3169.35	31543.5	$^4D_3 -$ bP_2	
4	3194.58	31294.0	$^4D_2 -$	
3	3209.39	31149.6	$^4D_1 -$	
3	3565.82	28036.1	$^4S_2 -$	
1	2869.93	34833.8	$^4P'_3 -$	
1	2888.39	34611.2	$^4P'_2 -$	
2	3132.20	31917.2	$^4D_4 -$	
2	3165.68	31579.6	$^4D_3 -$ cP_3	
2	3190.86	31330.3	$^4D_2 -$	
4	3561.21	28072.4	$^4S_2 -$	
6	4428.56	22573.9	$lP -$	$nP_3 - P_2$

TABLE 5 (*Continued*).

1	2858.02	34979.0	$^4P'_3-$	dP_2	
3 dr.	2876.41	34755.5	$^4P'_2-$		
0	2891.50	34574.0	$^4P'_1-$		$^{4\prime}P'_2-bP_2$
4	3118.00	32062.6	$^4D_4-$		
2	3151.12	31725.6	$^4D_3-$		
3	3176.14	31475.8	$^4D_2-$		
2	3190.86	31330.1	$^4D_1-$		?
7	3542.89	28217.5	$^4S_2-$		?
4	2792.04	35805.8	$^4P'_3-$	eP_3	
4	2809.51	35582.9	$^4P'_2-$		
4	3039.62	32889.7	$^4D_4-$		?
3	2762.97	36182.3	$^4P'_3-$	fP_2	
2	2780.05	35959.9	$^4P'_2-$		
3	2794.22	35777.6	$^4P'_1-$		
3	3059.15	32679.3	$^4D_2-$		
2	3035.95	32929.0	$^4D_3-$		qP_2-P_2
1	3072.68	32535.4	$^4D_1-$		
1	3397.84	29422.0	$^4S_1-$		
3	2756.68	36264.8	$^4P'_2-$	gP_1	
1	2770.63	36082.2	$^4P'_1-$		
2	3030.82	32984.8	$^4D_2-$		
2	3044.10	32840.9	$^4D_1-$		
1	3362.90	29727.7	$^4S_2-$		$^4D_2-{}^4D'_1$
3	4217.15	23706.0	$^4D'_4-$	nP_3	
5	4231.61	23625.0	$^4D'_3-$		
0	4565.3	21898.2	aP_2-		
0	4574.4	21854.7	bP_2-		?
1	4612.78	21672.8	dP_2-		
0	4795.7	20846.2	eP_3-		

TABLE 5 (*Continued*).

0	4205.60	23771.1	$^4D'_3$ —	mP_2	
1	4224.57	23664.4	$^4D'_2$ —		
0	4242.19	23566.8	$^4D'_1$ —		
2	4535.35	22042.8	aP_2 —		
0	4544.1	22000.4	bP_2 —		
0	4849.4	20615.0	fP_2 —		
0	4922.3	20310.0	gP_1 —		
2	4062.89	24606.1	$^4D'_3$ —	oP_3	
1	4080.44	24500.2	$^4D'_2$ —		
3	4369.79	22877.9	aP_2 —		
2	4377.95	22835.3	bP_2 —		
2	4384.98	22798.3	cP_3 —		
4	4413.16	22653.1	dP_2 —		
2	4580.36	21826.2	eP_3 —		
0	2972.28	33634.4	$^4D'_4$ —	pP_3	
0	2979.5	33552.9	$^4D'_3$ —		
1	2988.93	33447.0	$^4D'_2$ —		$^4D'_1 - cP_1$
3	3141.38	31829.9	aP_2 —		?
0	2976.65	33585.0	$^4D'_3$ —	qP_2	
0	2986.08	33478.9	$^4D'_2$ —		
0	2994.90	33380.4	$^4D'_1$ —		
0	2980.2	33545.0	$^4D'_2$ —	rP_1'	
1	2988.93	33447.0	$^4D'_1$ —		$aP_2 - pP_3$
0	4086.69	24462.7	$^4D'_4$ —	lP	
0	4100.27	24381.7	$^4D'_3$ —		
0	4118.11	24276.1	$^4D'_2$ —		
1	4627.93	21601.9	eP_3 —		
1	4322.66	23127.4	xP_2 —		
4	4413.16	22653.1	aP_2 —		$dP_2 - oP_3$
4	4421.38	22611.4	bP_2 —		

TABLE 5 *(Continued).*

1	2933.71	34076.6	$^4P'_3$ —	P_2	
1	2953.10	33852.8	$^4P'_2$ —		
2	3243.42	30822.9	$^4D_3$ —		
3	3269.84	30573.7	$^4D_2$ —		
2	3659.91	27315.3	$^4S_2$ —		
0	4399.95	22720.8	mP_2 —		
6	4428.56	22573.9	nP_3 —		$cP_3 — lP$
0	4244.17	23555.1	oP_3 —		
0	3075.73	32502.8	pP_3 —		
1	3072.68	32535.4	qP_2 —		
1	2916.22	34280.9	$^4P'_3$ —	xP_2	
0	2935.29	34058.2	$^4P'_2$ —		
1	2951.15	33875.8	$^4P'_1$ —		
3 dr.	3248.16	30777.8	$^4D_2$ —		
2	3263.40	30634.1	$^4D_1$ —		
1	3632.74	27520.6	$^4S_2$ —		
1	4439.95	22516.5	mP_2 —		
2	4468.86	22370.8	nP_3 —		
7	3829.75	26103.9	P_2 —	T	
4	3799.99	26308.4	xP_2 —		
9	3727.09	26823.0	bP_2 —		
2	3721.87	26860.6	cP_3 —		
3	3701.79	27006.3	dP_2 —		
0	3592.0	27831.7	eP_3 —	?	
1	3121.57	32025.9	$^4D_2$ —	U	
1	3135.79	31880.7	$^4D_1$ —		
0	3475.40	28765.5	$^4S_2$ —		
4	3628.09	27554.8	T_2 —		
1	4732.5	21124.6	nP_3 —		
0	4719.2	21184.1	kP —		
0	4699.6	21272.5	mP_2 —		
5	4569.02	21880.4	lP_2 —		
4	4522.65	22104.7	oP_3 —		

TABLE 5 *(Continued).*

1	4206.43	23766.4	$^4D'_4$—	kP
2	4220.89	23685.0	$^4D'_3$—	
2	4240.10	23577.7	$^4D'_2$—	
3	4257.82	23479.6	$^4D'_1$—	
2	4416.77	22634.6	P_2—	
4	4457.04	22430.1	xP_2—	
2	4553.21	21956.4	aP_2—	
0	4561.9	21914.6	bP_2—	
0	4600.16	21732.3	dP_2—	
0	4781.9	20906.4	eP_3—	
1	3154.80	31688.6	$^4D_4$—	yP_3
3	3188.73	31351.4	$^4D_3$—	
5	3214.33	31101.7	$^4D_2$—	
0	3590.47	27843.6	$^4S_2$—	
4	3753.79	26632.2	T—	
1	4534.55	22046.7	nP_3 -	
1	4384.08	22803.4	lP_3—	
2	4341.42	23027.4	oP—	
0	3123.2 ?	32009.2	qP—	
3	4133.65	24184.9	$^4D'_2$—	R_1
3	4150.65	24085.7	$^4D'_1$—	
2	4339.76	23036.2	xP_2—	
4	4430.91	22562.4	aP_2—	
3	4439.30	22519.8	bP_2—	
2	4446.43	22483.7	cP_3—	
1	4475.34	22338.4	dP_2—	
2	4588.11	21789.3	U—	
0	4647.4	21511.4	eP_3—	
0	4730.1	21135.3	fP_2—	

Comparison with the F I spectrum.

For determination of more exact values of terms it will be necessary to know a series relation. However, it will be interesting to compare the order of the analogous terms of which the theoretical interpretation is sure.

In both spectra the $^4P'$ term lies in contrary to the experimental rule of HUND deeper than the ^4D term. The interval ratio in the $^4P'$ terms has a value which not agrees with the value called for by LANDE's intervalrule.

TABLE 6.

F I				*Ne II*		Interval ratio		
Term	*j*	Term-value (relative)	Term-difference	Termvalue (relative)	Term-difference	LANDÉ	*F I*	*Ne II*
^2P	2	135320	407	±270000	[780]			
	1	134913		269220				
		(16.6 Volt)		(33.4 Volt)				
^4P	3	58617.0	274.7	117000.0	518.0			
	2	58342.3	160.0	116482.0	299.0	1.67	1.72	1.73
	1	58182.3		116193.0				
$^4P'$	3	45104.8	122.9	89938.2	222.6			
	2	44981.9	102.7	89715.6	182.5	1.67	1.19	1.21
	1	44879.2		89533.1				
^4D	4	44035.4	176.6	87022.7	337.8			
	3	43858.8	144.5	86484.9	249.7	2.33	2.12	2.34
	2	43714.3	83.4	86435.2	144.1	1.67	1.73	1.73
	1	43630.9		86291.1				
^4S	2	42595.0		83178.0				

Summary.

A number of 180 lines of the ionized *Neon (Ne II)* has been classified in a termscheme. The deepest quartet terms are identified. The *Ne II* spectrum has an analogous structure as the *F I* spectrum. The analogous terms have in *Ne II* approximately the double value as in *F I*.

Laboratory "Physica" of the University
or Amsterdam.

May 1927.

Physics. — *On the liberation of electrons from a metal surface by positive ions.* By F. M. PENNING. (Communicated by Dr. G. HOLST.)

(Communicated at the meeting of October 29, 1927).

§ 1. The work to be described in the following pages was undertaken on the suggestion of Dr. HOLST, in order to get more information about the part played by the positive ions in a gas discharge. According to the theory of TOWNSEND [1]) new electrons are formed in the bulk of the gas as a consequence of the ionising action of the positive ions; according to another view it is the bombardment of the cathode by positive ions of rather high velocity which gives rise to the liberation of these electrons. Several years ago a new theory was developed by HOLST and OOSTERHUIS [2]). This theory supposes that positive ions, which strike without any appreciable velocity a suitable metal surface, are able to extract electrons from this surface (of course besides those which are required for neutralisation). The condition for this happening is $V_i > 2\varphi$ ($V_i =$ ionising potential of the gas, $\varphi =$ work function of RICHARDSON). This theory was suggested by the phenomenon of the "negative striations" which HOLST and OOSTERHUIS observed in Neon of about 10 mm pressure at low currents (about 1 μA) [3]). From the facts that these striations had a sharp boundary towards the cathode side and that the layer immediately before the cathode was absolutely dark, it was concluded that the electrons were liberated from the cathode surface and not from atoms in the bulk of the gas [4]). Moreover, the velocity of the positive ions in this case must be very small, as a consequence of the small value of the mean free path.

In this connection we shall mention an analogous phenomenon which was observed in this laboratory by G. HERTZ [5]). In a tube as shown in fig. 1. with a heating filament K and an anode A a Neon low voltage arc is burning (pressure about 1 cm). When the plate P is held at a negative potential with respect to A, it attracts positive ions (which can diffuse through the gauze). Now between P and A striations are observed of the same kind as the above-said. Obviously these striations are caused by electrons which are liberated from P by positive ions and which excite the neon atoms on their way to A. The first layer is observed at a potential difference of about 20 V between P and A. Under these circumstances the potential difference along the distance of a mean free path for a pos. ion is only 0.01 V; clearly in this case there can be no question of a liberation of electrons by the bombardment of high velocity ions.

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[1]) J. S. TOWNSEND, Electricity in gases, 1915.

[2]) G. HOLST and E. OOSTERHUIS, Physica 1, 78, 1921; Comptes Rendus **175**, 577, 1922; Phil. Mag. **46**, 1117, 1923.

[3]) l.c.

[4]) This may be concluded also from the fact that the material of the cathode has great influence on the sparking potential, see G. HOLST and E. OOSTERHUIS, Versl. Kon. Ak. v. Wet. Amsterdam **29**, 849, 1920.

[5]) Unpublished.

It seems possible, however, that in the experiment just described the electrons were liberated by radiation. In order to investigate this point, another discharge tube was used as designed in fig. 2. Instead of one plate there were two plates P_1 and P_2 mounted in the same way within two glass tubes G_1 and G_2. The radiation from the main discharge may reach directly P_1 but it cannot reach P_2. However, the striations show themselves in about the same manner before P_1, as before P_2; therefore we may conclude that the radiation cannot have much influence.

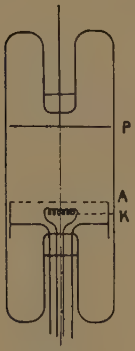

Fig. 1.

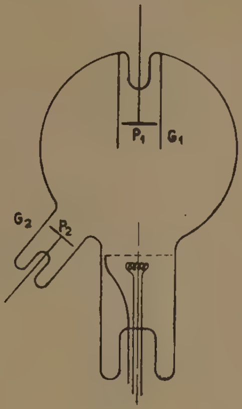

Fig. 2.

§ 2. We have tried to determine experimentally how many positive ions (especially neon ions 1)) of zero velocity are required to liberate one electron from a metal surface. We should, however, mention immediately that up to the present only preliminary results have been obtained, as a consequence of the difficulties of these experiments.

For the first experiments a tube was used of the type shown in fig. 1, but with a gauze G and a plate M instead of the plate P alone (fig. 3). In order to get a larger supply of ions, the anode A was placed under the cathode K. When the gauze is brought at a negative potential with respect to K, it will be struck only by pos. ions; on the other hand, when M is brought at a positive potential with respect to A, it will gather only electrons. In order that no electrons from the main discharge may reach M, the holes of the gauze should be small 2); smallness of the holes on the

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1) Unless otherwise mentioned the following experiments are made with neon.

2) The gauze used had 24 threads of 0.15 mm diameter at 1 cm in each direction.

other hand gives rise to the inconvenience that only a fraction of the electrons from G reaches M.

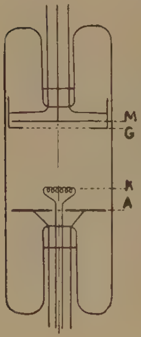

Fig. 3.

As our intention was to study the action of positive ions with low velocity, the values of the potentials in the tube should be chosen as small as possible. Therefore the gas-pressure should be high (several mm) and in that case the electrons, liberated from G, may ionise the gas atoms between M and G, in case the potential of M is raised. Therefore the current to M, plotted as a function of the potential of M, does not show saturation; distinct kinks are observed at the points where the potential has increased with V_i.

The electron current becomes more constant when the gas pressure is decreased, but then we must give up at the same time the advantage of the low potentials. When, moreover, the distance between M and G is taken as small as possible (about 1 mm), the current to M becomes nearly constant with increasing potential. The number of electrons, reaching M, is then 3 to 5 % of the number of positive ions which go to G (at pressures between 0.2 and 0.02 mm and pos. ion currents from 0.2 to 20 mA). Besides, the variation of the potential of G (with respect to the cathode) between —20 and —80 V had little influence on this percentage; so it is obvious that the number of liberated electrons is affected only to a small extent by the velocity of the positive ions.

It is not right, however, to conclude from these measurements that from 20 to 30 ions are required to extract one electron from the metal; on the contrary a smaller number will be sufficient, because the electrons, liberated from G, will move partly also in the direction to A.

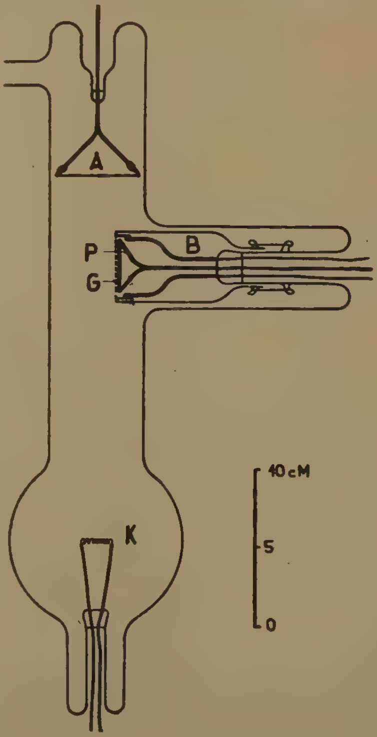

Fig. 4.

§ 3. For measurements of the same kind we used also a positive column in neon of low pressure (0.02 mm). The tube is shown in fig. 4. The discharge (some hundred mA) is maintained between the oxide cathode K and the anode A. The positive column contains a collector which, when brought at a negative potential, will gather pos. ions. In order to measure the number of electrons α, which is liberated on the average by one positive ion, we use again the method described in § 2. A side tube contains a gauze G (copper) and a plate P; the current to both these electrodes is measured. To protect the backside of the plate P from electrons, this plate is enclosed in a glasstube B, which is fastened to the glasswall. The results for a few series of measurements are shown in fig. 5 1). The potentials are given with respect to the anode (this will always be done in the following). The gas in the neighbourhood of the gauze G will be on about the same potential as the anode, therefore, on the moment that the positive ions strike the gauze, their velocity (measured in Volts) will have about the same value as the potential of G. Because it is necessary that the electrons from the main discharge should be stopped, the gauze cannot be brought on a much higher potential as —100 V (the potential of the cathode is in this measurement about —60 V). The probable trend of the curve above —100 V is dotted in fig. 5. The value of α for a velocity zero (α_0) is found to be about 2 % (even if the curve through the measured points should be extrapolated in a different way). As the gauze in this tube was of the same construction as that of § 2, these measurements are open to the same objection as mentioned there. From the measurements of this and the foregoing § we may conclude: 20 to 50 pos. ions (of zero velocity) at the utmost are required to liberate one electron from a Cu-surface.

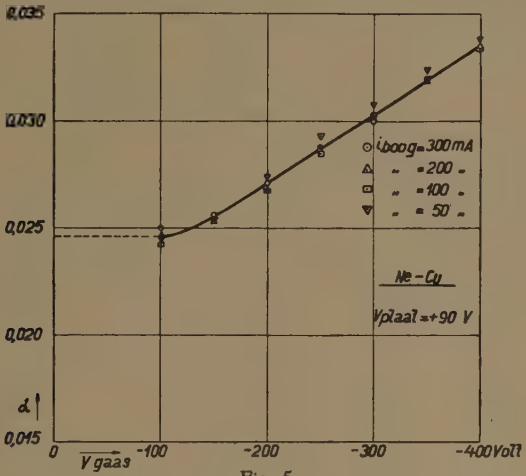

Fig. 5.
boog = arc, plaat = plate, gaas = gauze.

The same tube was used also for measurements with argon filling and

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1) The values of the electron currents are in this case from 0.1 to 0.01 mA.

with *Mg* instead of *Cu* as electrode material. As should be expected in connection with the values of V_i and φ, the value of α was for *Ar* < for *Ne* and for *Mg* > for *Cu*.

§ 4. To meet several drawbacks of the described measurements, afterwards another method was followed. It was desirable to study the action on the pos. ions in a space of very low pressure, where the collisions between positive ions (or secundary electrons) and atoms could be neglected. When the gas pressure in the main discharge is lowered under 0.02 mm the arc potential is rising rapidly. This should be prevented, as we shall study the action of pos. ions of low velocity. Therefore it is necessary to bring the pos. ions in a room where the gas is pumped away and which is communicating only through a narrow canal with the main discharge. After experiments with tubes of somewhat other constructions the tube of fig. 6 gave the best results (tube II) [1]).

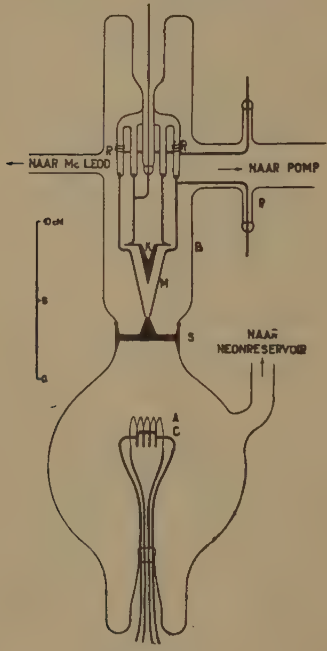

Fig. 6.

The cathode *C* is an oxide filament, the anode a large tungsten spiral which could be degassed by glowing. In the sidewall of the bulb containing the main discharge, a chromic iron piece *S* is molten and to this again the tube *B* with the collector system. This system contains an iron cone *K* which is used as a collector for the pos. ions; the cone is surrounded by a mantle *M* which collects the secondary electrons. *S* is perforated by a canal of 1 mm diameter and 15 mm length. *B* is connected by a short, wide tube with a diffusion pump which keeps the gas-pressure on a value of about 0.0002 mm (with a pressure of 0.02 mm in the bulb). The pressure at both sides can be measured with a Mc-Leod gage. As in these experiments the gas is rapidly disappearing by metalsputtering, the bulb is connected with a large reservoir of 10 l capacity; in this way many measurements may be made before it is necessary to admit fresh gas. The discharge tube is separated by tubes in liquid air from parts of the apparatus containing mercury or stopcocks.

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[1]) In the tube I, used before, the anode was mounted at a much larger distance from the cathode, so that a positive column occurred. The chromic iron piece *S* was molten in the side wall of the tube, containing the positive column. The canal in *S* was 20 mm. ong in this tube. The arrangement of tube II was chosen afterwards to lower the arc potential.

As only a small number of pos. ions is reaching the cone *K* the currents are very small (unto $< 10^{-9}$ A); therefore the electric insulation should be very careful. The leads-in of *M* and *K* are molten in the glass, at a large distance of each other. Moreover, the leads-in of *M* and the glass-bars in the tube which support *M* are surrounded by conducting guard rings *R* which are helt on the same potential as *M*. In the same way the points of support of the connecting wires, of the galvanometer and so on are protected from leakage.

Before the measurements the tube was pumped at a temperature of about 400° C. Afterwards the metal parts were degassed; the iron cone could be glown in vacuum by means of high frequency currents.

As it is impossible to measure directly with this arrangement the effect of positive ions with zero velocity, it was tried to measure the percentage of liberated electrons (α) as a function of the velocity of the pos. ions, and to extrapolate to a velocity 0. The measurements were made in the following way: the canal in *S* and the mantle *M* were helt on the same potential, e.g. —100 V with respect to the anode (the arc potential is always < 100 V). The pos. ions arrive in the part of low pressure with a velocity between 0 and 100 V. Between *S* and *K* a potential difference of 50—900 V is put, which accelerates the pos. ions; as the pos. ions make nearly no collisions in this part of the tube (mean free path about 50 cm) they arrive at the cone with a velocity varying from 0—100 to 900—1000 V. The liberated electrons are moving towards *M*; indeed, although the end of the canal is at the same potential as *M*, the direction of the field is such that the percentage of the electrons which is going to *S* may be neglected. Moreover as a consequence of the relative positions of *K* and *M* there is a strong electric field at the surface of *K*, so that we may expect a rapid saturation of the electron current. As a matter of fact with this collector system saturation was reached already at a potential difference of about 20 V between *K* and *M*, whereas with a system used before, this was only the case at 200 V. To show this, in Fig. 7 are plotted the current to the mantle *M* and the cone *K* for tube I (collector system the same as in tube II [1]) with the following potentials:

Canal (= kanaal): —150 V.
Cone (= kegel): —500 V.
Mantle (= mantel): varied.

(All these potentials are measured as before with respect to the anode.) An electron current in the direction from cone to mantle is taken as negative for both electrodes; so in Fig. 7 for potentials above —500 V pos. ions are going to the cone and electrons to the mantle. In case

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[1] Compare the footnote on p. 18.

$V_{\text{mantle}} = V_{\text{canal}}$ the positive ions which energe from the canal will not be retarded ; when the mantle is made more negative, the ion current to the cone remains constant ; on the contrary, when the mantle becomes positive with respect to the canal the ions will be partly stopped and the current to the cone becomes smaller. For potentials of the mantle between —150 and —480 V the electron current and the pos. ion current remain nearly constant, showing a good saturation. When $V_{\text{mantle}} = V_{\text{cone}}$ the current of the mantle becomes positive instead of negative, as should be expected. The small pos. ion current to the mantle at first increases slowly but then again very rapidly and when $V_{\text{mantle}} = -1000$ V the mantle and the cone have interchanged their functions. The value of a which is deduced from the measurements at —1000 V is larger as that found in case the positive ions are collected by the cone, but in the first case the velocity of the ions is twice as large.

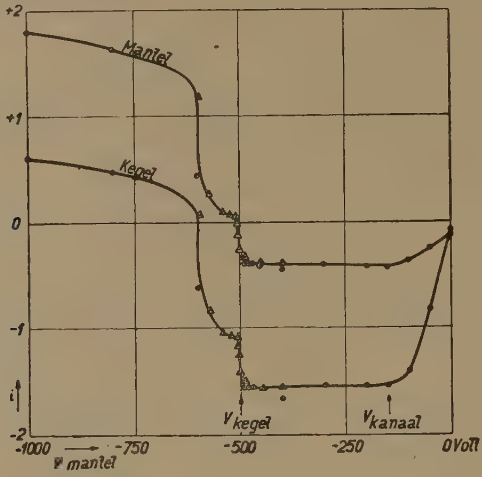

Fig. 7.

Concerning the final measurements the following may be remarked. The currents were measured with a galvanometer (sensibility 0.3×10^{-10} A); in order to use the same instrument for both currents, a throw-over switch was applied, consisting of mercury cups in paraffine ; to avoid leaking currents each mercury cup was surrounded by a conducting ring helt at the same potential, in the way mentioned at the description of the tube II. With the aid of such precautions we succeeded in reducing the leaking currents to a negligeable amount. The electrodes were connected to the switch in such a way that the galvanometer deflections had the same direction for both currents, and further by suitable shunts, the order of magnitude of the deflections was made the same, in order to diminish the influence of the zero point of the galvanometer.

§ 5. The results of a few series of measurements with the tubes I and II are shown in Fig. 8. Every series was measured in the direction ot increasing and of decreasing voltages (the corresponding points are designed as circles and triangles). As ordinate is plotted a: the number of electrons liberated on the average by one pos. ion. As the current to the cone consists of the sum of the pos. ions which reach it and the electrons which leave it:

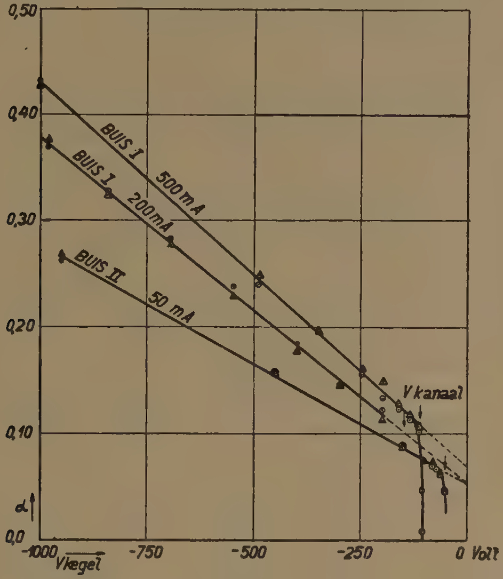

Fig. 8.
Buis = tube, Kegel = cone, Kanaal = canal.

$$a = \frac{| i_{\text{mantle}} |}{| i_{\text{cone}} | - | i_{\text{mantle}} |}.$$

As abscissa is plotted the potential of the cone; this gives at the same time, within the limits already mentioned higher up, the velocity of the pos. ions (measured in Volts). The potential of the canal should of course be lower than the lowest point of the cathode; the mantle is on the same potential as the canal. As soon as the potential difference between the cone and the mantle becomes lower than about 20 V, the mantle will no longer collect all the liberated electrons (see Fig. 7); the values, found for a, are then too low. Consequently, the curves in Fig. 8 are then deviating from the original direction; when $V_{\text{mantle}} = V_{\text{cone}}$ they go down steeply. There is, however, no reason to suppose that the real value of a also should vary suddenly; on the contrary always when in consequence of a lower potential difference between the cathode and the anode, a lower value could be chosen for V_{canal}, the curve bent off at a higher value of V_{mantle}. It is, therefore, very probable that we may extrapolate the curves of Fig. 8 in the manner as shown by the dotted lines and that for iron a_0 (the value of a for pos. ions with zero velocity) is about 0.06 1).

The values of a_0 in Fig. 8 are diverging between 0.05 and 0.07. As a matter of fact in other series (measured partly with tubes of a somewhat other type) even more differing values were found, viz. unto 0.12 ; likewise the value of a for larger ion velocities was not constant ; the value for 1000 V (a_{1000}) varied, e.g. with tube I in the course of the measurements (which extended over 19 days) between 0.44 and 0.27. Probably the condition of the metal surface has a great influence, as is apparent also in photoelectric researches 2). We intend to make further researches with regard to this point. Anyhow it could be stated that immediately after the glowing of the metal a_{1000} was smaller viz. 0.23—0.30 ; afterwards the value was increasing again gradually.

Though the value of a is not yet known with sufficient accuracy, we may conclude from the above-mentioned experiments (also in connection with the results of § 2 and § 3) : Neon ions which reach without velocity a copper or iron surface are able to liberate electrons from this surface, and the value of a_0 is about 5 to 10 %.

§ 6. Finally, we shall try to answer the question how far the obtained results are in agreement with those of other authors. Many experiments of this type have been done with positive ions which are emitted in vacuum by heated aluminium phosphate and similar substances. We also tried at first in this way to get some information about the liberating of electrons by positive ions. The result was, that ions of this type with zero velocity did not cause any measurable electron emission from a Ni-surface 3). At a velocity of 100 V, in our experiments a was still < 0.01. However, we have in this case ions of *Na* 4) or other alkali metals with small V_i, and one should expect that an extra electron can be liberated

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1) Recently, by means of another apparatus, the measurements could be extended unto ion velocities of less than 7 Volts. In this case also the value of α was still a few percents, see Physica **8**, 13, 1928. (*Note added in the translation*).

2) See e.g. LEE A. DU BRIDGE, Phys. Rev. **29**, 451, 1927. This writer measured the total photoelectric current from a Pt surface. This was glown several times at 1250° C.; the constant limit value was reached only after 50, sometimes only after 150 hours glowing; after 20 hours glowing e.g. the measured value was still $2^1/_2 \times$ the limit value.

3) The same result was obtained by N. CAMPBELL, Phil. Mag. **29**, 783, 1915; W. L. CHENEY, Phys. Rev. (2) **10**, 325, 1917; A. L. KLEIN, Phys. Rev. (2) **26**, 800, 1925; W. F. JACKSON, Phys. Rev. (2) **28**, 524, 1927. Compare also: E. BADAREU, Phys. Zts. **25**, 137, 1924. The numerical values for α at higher velocities show very large differences.

4) See e.g. O. W. RICHARDSON, The emission of electricity from hot bodies, 1921. By measuring the current in a diode (with a hot anode, covered with aluminium phosphate) as a function of the potential, we could show that the ions in the above-said experiments were Na-ions.

only by an ion when $V_i > \varphi$. Now this condition is not fulfilled for the ions used and for metals as *Fe*.

With gas ions (H_2) experiments have been done by BAERWALD. This physicist also used a tube which was devided into two parts by a canal. This canal was used as a cathode of a glow discharge; therefore the canalrays which entered the low pressure part of the tube always had a rather large velocity. As collector metal brass was used. In order to compare our method with that of BAERWALD [1]), we performed also some measurements on hydrogen. A tube of type I was used, but with a somewhat different collector-system. However, we found with ions of 1000 V velocity a value of *a* which was only 10 % of that found by BAERWALD. This must perhaps partly be ascribed to the different manner in which the pos. ions are formed (molecule- and atom ions?); also we should make allowance for the circumstance that BAERWALD's apparatus could not be pumped at higher temperature and that it could not be degassed; finally perhaps brass gives in this respect quite other results as *Cu* or *Fe*.

It is a pleasure for the writer to thank Mr. J. C. DOUZE for his assistance in the measurements and in the constructing of the tubes.

Natuurkundig Laboratorium der
N.V. Philips' Gloeilampenfabrieken.
Eindhoven, October 28th, 1928.

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[1]) H. BAERWALD, Ann. d. Phys, **65**, 167, 1921.

Anatomy. — *A cutaneous branch of the facial nerve in a teleost.* By Dr. C. J. VAN DER HORST. (Central Institute of Brain Research. Amsterdam.) (Communicated by Dr. C. U. ARIËNS KAPPERS.)

(Communicated at the meeting of December 17, 1927).

The branchial nerves of lower vertebrates are built up from several components viz., motor fibers, gustatory fibers, fibers for the general sensibility of the mucous membrane in mouth and throat, and fibers for the general sensibility of the skin (HERRICK, JOHNSTON).

Concerning different nerves (e.g. facial, vagus, etc.), the contribution of the different sensory components is very variable. Thus the trigeminus consists for the greater part of fibers for general sensibility of the skin, a smaller part finds its endings in the epithelium of the mouth cavity, whereas gustatory fibers are absent.

In this point the trigeminus differs considerably from the three following branchial nerves: facialis, glossopharyngeus and vagus. These three nerves consist for the greater part of gustatory fibers and fibers of general sensibility to the mucous membrane, whereas only very few fibers, or none at all, have free endings in the skin of the head.

As far as is known a cutaneous branch of the vagus is always present, though it is sometimes so small that it is hardly visible. In some cases (*Prionotus* after HERRICK) this vagus branch is very large.

Cutaneous fibers seem only seldom to be present in the glossopharyngeus. EWART found them in selachians, HOUSER mentions them in *Mustelus canis*. On the other hand NORRIS and HUGHES did not find them in *Acanthias*. According to COLE cutaneous fibers of the glossopharyngeus occur in *Chimaera* and according to JOHNSTON in *Petromyzon* and *Acipenser*.

More often than in the glossopharyngeus, cutaneous fibers are present in the facial nerve. According to JOHNSTON, the ramus hyomandibularis VII of *Petromyzon dorsatus* contains a considerable number of cutaneous fibers that join centrally the spinal trigeminus tract. Whereas the cutaneous branches of the glossopharyngeus and vagus have their endings on both the dorsal and ventral aspects of the head, those of the facialis in *Petromyzon* are limited to the ventral side.

KAPPERS was able to demonstrate a cutaneous branch of the facialis, at least the central part of it, in *Heptanchus* and *Hexanchus*. Compared with the spinal fifth, the descending facialis in these sharks is only a very small bundle, which soon after its entrance, joins the trigeminus. In *Chlamydoselache* MERRIT HAWKES also found some small branches of the facialis going to the skin of the ventro-lateral side of the head. On the other hand cutaneous fibers are wanting in *Acanthias*, according to NORRIS and HUGHES, in the roots as well as in the peripheral branches of the facialis.

In *Chondrostei* NORRIS found no trace of a somatosensory element in the facialis roots. ALLIS described a cutaneous branch in *Polyodon*, but NORRIS is of the opinion that it is a lateralis branch. And yet the truncus hyomandibularis receives cutaneous fibers, but by way of an anastomosis with the vagus. This connection has already been found before by STANNIUS in *Acipenser*. ALLIS showed its presence in *Polyodon*. NORRIS could state its somato-sensory character without doubt in *Polyodon* and *Acipenser*. In *Scaphirhynchus* this was less clear.

In Amphibians a connection exists between the vagus and the facialis in the same way. In *Siren lacertina*, however, this anastomosis is very thin (DRÜNER); sometimes it may be wanting altogether. NORRIS was able to show that in this animal, near the exit of the motor facialis, cutaneous fibers split off from the very superficially located spinal trigeminus and go out with the facialis. The ganglion of these cutaneous fibers is situated at the ventral side of the motor facialis. RÖTHIG also demonstrated such a cutaneous branch belonging to the facialis in *Megalobatrachus* and *Bufo*.

In teleosts the truncus hyomandibularis contains cutaneous fibers, as was demonstrated by RUTKIEWICZ in *Ameiurus* and by HERRICK in *Ameiurus*, *Menidia* and *Gadus*, but these cutaneous fibers unite only peripherally with the facialis. They leave the brain with the spinal trigeminus and their cellbodies form a part of the ganglion Gasseri. A real cutaneous branch of the facialis has not been shown in teleosts before the present finding.

Somato-sensory fibers were found by KINGSBURY in *Amia*; this was confirmed by NORRIS. The latter author could point out the presence of these fibers in even a greater number in *Lepidosteus*. This somato-sensory bundle consists almost entirely of fibers without myelin sheath. They split off from the spinal trigeminus which runs quite near the outer surface of the oblongata and their ganglion is situated at the caudal and ventral side of the whole complex of the trigeminus and facialis ganglia. The fibers run in the truncus hyomandibularis to the periphery.

As mentioned above a cutaneous branch of the facialis has not been found in teleosts before, but in *Albula* such a branch is present. Although the fibers are unmyelinated for the greater part, they form such an apparent and well defined bundle that it is the most striking one in the medulla oblongata, especially in the region of the vagus, and this bundle of the descending facialis can be traced from its entrance to its central ending without any difficulty.

In the intracerebral path, the cutaneous branch of the facialis in *Albula* differs considerably from the cutaneous branches of the other above mentioned animals. In the latter the somato-sensory facialis joins the descending trigeminus directly after its entrance. In *Amia* and *Lepidosteus* the trigeminus bundle runs near the very surface of the oblongata, according to NORRIS, and the same holds true in *Siren lacertina*. But in *Albula*, as in all other teleosts, the descending trigeminus is situated deeper in the oblongata at the dorsal side of the secundary gustatory tract.

In selachians, the trigeminus bundle has the same position and thus the cutaneous branch of the facialis of *Hexanchus* and *Heptanchus* penetrates deeply in the oblongata to join the trigeminus, as was shown by KAPPERS. On the other hand, in *Albula* the descending facialis remains near the periphery and a union with the trigeminus does not occur. Only near the end nucleus behind the calamus scriptorius does the trigeminus approach the surface and the bundles run alongside each other.

The cutaneous branches of the vagus, which are very apparent in *Albula*, no longer unite with the descending trigeminus in this teleost, though in others they do. In *Albula* these branches unite with the descending facialis. A cutaneous branch of the glossopharyngeus is not present.

From its entrance into the oblongata, the spinal facialis runs caudad in the ventral part of the lateralis nucleus near the outer surface. With the diminishing size of the lobus liniae lateralis, the descending facialis approaches the dorsal side of the oblongata more and more. In the vagus-region the spinal trigeminus also comes near the surface. These two descending bundles are separated here by the entering sensory vagus roots.

The cutaneous fibers of the vagus join the facialis directly at its ventral side. A little behind the calamus scriptorius after the entrance of the last sensory vagus roots, the descending trigeminus takes a position at the medial side of the facialis and vagus bundles.

Whereas the visceral commissural nucleus is rather small, the somatic one is large. At its lateral side and hence caudad, this somatic commissural nucleus forms one mass with the endnucleus of the descending trigeminus, facialis and vagus bundles and further on with the somato-sensory area of the spinal cord. Around this nucleus, the three descending bundles are arranged in such a way that the facialis is at its dorsal, the vagus at its lateral and the trigeminus at its ventral side. So the fibers penetrate the nucleus from all sides (fig. 1).

The dorsal spinal roots split up in two parts at their entrance. Of the first root smaller numbers of fibers curve dorsad and, running near the surface, join the vagus and the facialis. Greater numbers of fibers enter horizontally and join the trigeminus. Of the second spinal root, most of the fibers bend dorsad at their entrance.

Frontally, about at the level of the first dorsal spinal root, the somato-sensory nucleus is so large that it occupies nearly half the spinal cord (fig. 1). At the level of the second root, where nearly all facialis and vagus fibers seem to have disappeared, the nucleus is considerably smaller. And yet still further caudad the somato-sensory area remains very large in comparison with other teleosts. Here the nucleus exhibits the peculiarity that it does not form a more or less compact mass as is usually the case in teleosts, but has the shape of a rather thin curved lamella (fig. 2). Thus it assumes a form similar to the lower olive and other, especially sensory, areas that enlarge their surface by making folds. These lamellations not situated at the outer surface of the central nervous system are called by KAPPERS „inner cortical structures".

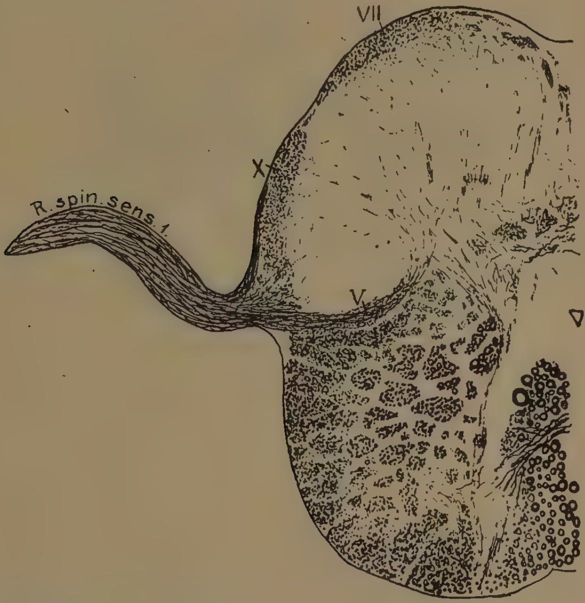

Fig. 1. Transverse section of the spinal cord of *Albula vulpes* at the level of the first sensory spinal root.

At its entrance into the medulla oblongata, the cutaneous branch of the facialis is situated rather at a distance from the gustatory branch (fig. 3). The latter runs more dorsally at the fronto-medial side of the nerveus lateralis anterior. Ventral to the lateralis is the entrance of the nervus octavus and here the fibers of this nerve surround the cutaneous branch of the facial from all sides. At its entrance, frontal to the nervus octavus, the descending facialis is in close relation with the motor seventh. The fibers of the latter surround those of the former almost entirely. According to NORRIS, in *Amia* and *Lepidosteus* also, the cutaneous branch of the facialis enters the oblongata in close proximity to the motor part and in *Siren* the motor root surrounds the cutaneous branch much in the same way as in *Albula*.

Somewhat nearer the periphery, these two facialis roots join the nervus lateralis anterior at its ventral side. The lightly coloured fibers of the cutaneous branch are separated from the lateralis by the heavily myelinated and thus, by the WEIGERT-method, darkly coloured motor fibers.

To this complex, the gustatory bundle of the facialis is added and, more frontally, also the trigeminus. The ganglion of the cutaneous root of the

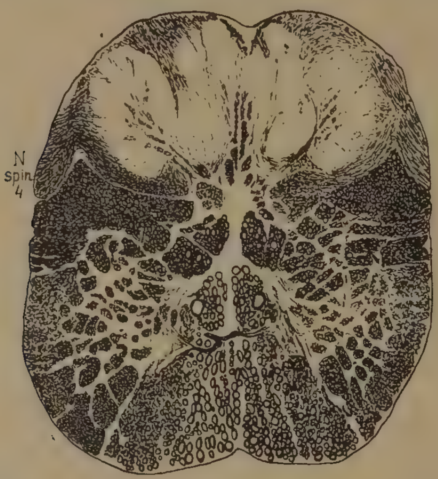

Fig. 2. Transverse section of the spinal cord of *Albula vulpes* at the level of the fourth spinal nerve.

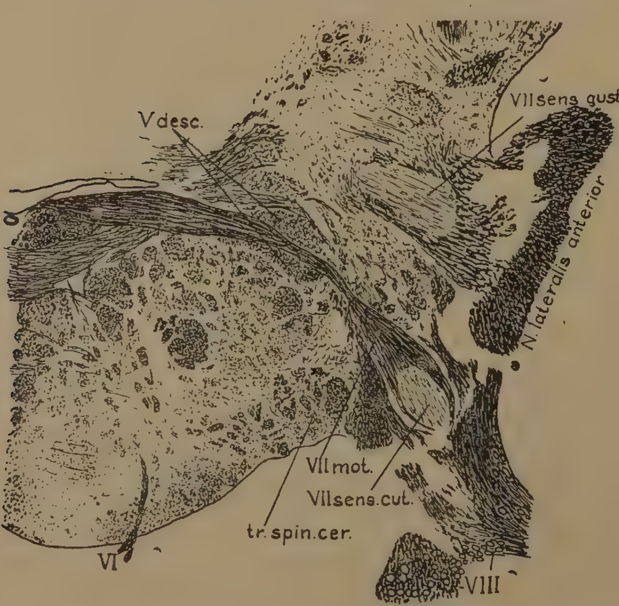

Fig. 3. Transverse section of a part of the medulla oblongata of *Albula vulpes* near the entrance of the N. facialis.

facialis is situated wholly within the skull cavity in contrast to the other ganglia of this complex. The cellbodies are much smaller than those of the lateralis. The cutaneous branch of the facialis goes to the periphery in the truncus hyomandibularis, as is always the case if such a branch is present.

I have studied various other teleosts in search of a cutaneous branch of the facialis. Nowhere have I found it. In *Megalops,* like *Albula* a very primitive teleost, it certainly is absent. But it is possible that a cutaneous facialis is present in the *Mormyridae*. The tractus spino-cerebellaris dorsalis that is highly developed in this family of fishes runs quite near the surface, as is also shown in the figures of STENDELL. So this tract in *Mormyridae* does not follow the ordinary way in close contact with the descending trigeminus, but runs nearer the surface and in the position where a descending facialis could be expected. It is possible that this tract contains facial fibers ; my material, however, was not sufficient to decide this and neither do the figures of STENDELL. In *Gadus* and notably in *Lota*, a cutaneous facialis is absent, though the sensibility of the skin of the head is highly developed in these fishes, and in relation with this, the cutaneous branches of the trigeminus and vagus are very large. Also in other teleosts the truncus hyomandibularis contains fibers for general sensibility of the skin that arise from the ganglion Gasseri, according to HERRICK, and so are trigeminus fibers that only join the truncus peripherally. It is possible that the cutaneous fibers going out in *Albula*, *Amia* and *Lepidosteus* with the motor facialis, leave the central nervous system in other fishes farther frontally, together with the cutaneous fibers of the trigeminus.

No doubt the sensibility of the skin of the head is highly developed in *Albula*, as has already been indicated by the extension of the somato-sensory area in the frontal part of the spinal cord. This extension is not caused by a hypertrophy of the dorsal spinal roots as in *Trigla* and *Prionotus* ; on the contrary these roots are rather small in *Albula*. And yet the descending trigeminus is not especially large in *Albula* ; in *Gadidae* it is much larger.

This makes it apparent that the same cutaneous fibers that enter with the trigeminus in nearly all teleosts, reach the central nervous system in *Albula*, *Amia* and *Lepidosteus* with the motor facialis. This shows some similarity with the phenomenon in motor roots that ADDENS calls "central anastomosis". The peculiarity of *Albula*, in contradistinction to *Amia* and *Lepidosteus*, is that the descending facialis does not join the spinal trigeminus, but remains as a well defined, separate bundle along its whole course.

According to GILL, *Albula* is met with especially near the coast, where it looks for its foods in the shallow water. The food consists of molluscs, especially lamellibranchiates, that live in the mud or sand and therefore are probably found by feeling. When the fish is looking for its food in the shallow water it has the head downwards and puts the tail out of the water. This habit may give an explanation for the highly developed sensibility of the head in *Albula*.

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(Communicated at the meeting of October 29, 1927).

§ 1. *Introduction, material and method.*

By BLAAUW the influence has been examined of various temperatures during flower-formation on the whole hyacinth. For this purpose the *late* variety Queen of the Blues was taken (BLAAUW, 1924).

Next however it was important to know how other varieties would behave. For this purpose we now examined the varieties l'Innocence and la Victoire *flowering early* in culture. In this case the bulbs were exposed only to those five temperatures for which most optima were found in Queen of the Blues, while we took care that sufficient room was left for possible deviating optima in these early hyacinths. Besides the results of these temperatures were only traced for the most important organs. This examination however is more extensive in so far, that the bulbs to be planted were not only exposed to an after-treatment in 13° C. after a preliminary treatment of 8 weeks in 23°, 25.5° and 28° C., but after a preliminary treatment of 3,5 and ca 8 weeks in 17°, 20°, 23°, 25.5° and 28° C.

To get an insight in the foundation and object of this experiment, for drawings, more detailed description and for comparison, see BLAAUW 1924.

At the beginning of July 1926 there were destined for this examination 210 + 640 bulbs of l'Innocence and 640 bulbs of la Victoire. Both varieties were so-called biennial of scooped material (= 3 year old) and selected for a circumference of 12—13 cms. The Queen of the Blues previously examined was a year younger and of a circumference of 7.5—9 cms. This should be specially borne in mind in judging flower-cluster-formation and number of flowers.

Of the 210 bulbs l'Innocence 10 bulbs were fixed on July 5, 1926 in alcohol 96 %, in order to trace the condition of the vegetation-point on the day on which the experiment was started. 200 bulbs were put in the temperatures 17°, 20°, 23°, 25.5° and 28°, viz. 40 in each temperature. Of these 40 bulbs groups of 10 bulbs were left in these temperatures for 3, 5, ca 8 and 12 weeks and next fixed, in order to trace in which temperature after a definite period the development had progressed farthest.

The 640 bulbs of l'Innocence and la Victoire were treated as has been explained with respect to the above 200 l'Innocence, but they were not fixed: after 3,5 and ca 8 weeks they were transferred from the above temperatures to 13° and 17° and planted after 12 weeks simultaneously with the groups of 20 which had been kept in 17° to 28° throughout the 12 weeks. For each of the 32 different combinations produced in this way 20 bulbs were taken. Since however the 20 bulbs l'Innocence which after 8 weeks should have gone from 17° to 13°, were kept in 17° bij mistake, this combination is left out; accordingly in the subjoined tables we find two groups with the treatment 17° for 12 weeks mentioned for l'Innocence.

§ 2. *Results of the temperature-treatment for the leaves.*

Number of foliage-leaves formed.

1. *At the beginning of the experiment.* The number of young foliage-leaflets on July 5, 1926 was 54 for the 10 bulbs examined. Of these 10 bulbs 7 had 5 leaflets, 2 bulbs 6 leaflets (in one of these bulbs one leaflet was not entirely split off) and 1 bulb 7 leaflets (one of which was not entirely split off).

2. *After the fixations.* After 12 weeks 200 bulbs had been treated in 20 various modes (see tab. 1) and fixed. The number of leaves in these 200 bulbs averagely amounted to 54.7 per 10, when those which are not entirely split off are also counted. Therefore some increase is again to be noted, just as in Queen of the Blues, but also with respect to l'Innocence we mean to be justified in drawing the conclusion that this slight *increase* is *of an accidental nature* and not due to the exposures to these temperatures.

TABLE 1. *l'Innocence.*

Number of young foliage-leaflets per 10 bulbs, when they have been in 17°—28° C., from 3 to 12 weeks.

	After 3 weeks	5 weeks	7½ weeks	12 weeks
17°	55	56	53	54
20°	53	53	54	54
23°	54	56	53	56
25.5°	54	53	57	58
28°	54	55	56	56

Is not this sufficiently clear from the fact that 54 is found in 25.5° after 3 weeks, 53 after 5 weeks, after 7½ weeks 57 and after 12 weeks 58. This would have to signify an increase, but this is impossible, for after 5 weeks the leaf-formation has long been finished and the flower-cluster-formation is in full swing.

Number of shooting leaves in spring.

In § 1 it has been said, that 640 bulbs of l'Innocence and of la Victoire were planted out after the various exposures. The tables 2 and 2*a* give their number of leaves, which in spring, when they had been pushed from the bulbs, could be counted. The recorded numbers have been counted on groups of 20 bulbs and converted per 10.

From these tables it appears, that (just as in Queen of the Blues) not all leaves grow out in lower temperatures.

TABLE 2. *l'Innocence.*

Number of assimilating leaves in the spring of 1927 per 10 bulbs.

		3 weeks		5 weeks		7½ weeks		12 weeks
Preliminary exposure	17°	34.0	—	35.5	—	—	—	46.5 and 47.0
	20°	39.5	48.0	47.5	52.5	51.1	50.0	54.0
	23°	51.0	52.5	53.2	53.7	53.5	54.5	54.5
	25.5°	46.5	54.2	53.0	54.5	55.5	53.5	53.0
	28°	52.0	55.0	54.0	56.5	54.5	56.5	55.5
After-exposure to:		13°	17°	13°	17°	13°	17°	—

TABLE 2*a*. *la Victoire*, as table 2.

		3 weeks		5 weeks		7½ weeks		12 weeks
Preliminary exposure	17°	56.5	—	61.6	—	61.5	—	64.0
	20°	59.5	60.0	61.5	62.6	61.0	62.0	62.5
	23°	61.5	59.0	62.0	59.5	61.1	60.5	60.5
	25.5°	60.5	61.0	60.0	60.5	61.5	62.3	62.0
	28°	62.0	62.0	61.5	61.5	62.5	62.5	64.0
After-exposure to:		13°	17°	13°	17°	13°	17°	—

In order to make all leaves grow out 13° is nearly always less favourable than 17°. Further it may be observed that the higher the temperature of the preliminary exposure, the shorter it need last to make all the leaves shoot. In l'Innocene 20° is necessary for 12 weeks, 25.5° and 28° but for 3 weeks. l'Innocence however can have a lower temperature than Queen of the Blues. All or nearly all the leaves already grow out in l'Innocence in 20° for 12 weeks, whereas in Queen of the Blues this does not occur before 25.5° for 8 weeks.

La Victoire (table 2*a*) deviates still more from Queen of the Blues; here there are no great differences to be indicated in the chosen temperatures. The complete number 64, which probably averagely amounts to 61 or 62, is here (with 64) already attained in 17° for at most 12 weeks, whereas the number 56.5 for the lowest temperature, viz. 3 weeks in 17° and next in 13°, compared with the two other varieties, deviates but little from the complete number. So in this respect la Victoire can stand 13° much better than l'Innocence.

Length of the foliage-leaves.

1. Table 3 gives the length of the young outer foliage-leaf measured at the peeled bulbs of the *fixed material*. The optima are printed fat. For Queen of the Blues and l'Innocence they are found in nearly the same treatment. The principal differences are, that for l'Innocence an optimum lies at ± 8 weeks 23°, whilst for Queen of the Blues this lies at ± 8 weeks 20° and further that for Queen of the Blues the leaves in 3 weeks 23°, 25.5° and 28° are equally long. Accordingly there is a striking correspondence. Here it is again corroborated, that according as the organ-*enlargement* (especially in September) is acting a more important part, as compared with the organ-*formation*, the optimum is shifted to a lower temperature.

TABLE 3. *l'Innocence.*
Average lenghts in mms of the outer foliage-leaves, still in embryonic condition ($n = 10$).

	Beginning July 5, '26	After 3 weeks	5 weeks	7 1/2 weeks	12 weeks
17°	3.69	4.72	6.45	10.30	**20.83**
20°	3.69	5.24	6.86	9.79	18.97
23°	3.69	5.23	**7.02**	**10.82**	18.12
25.5°	3.69	**5.47**	6.51	9.47	15.51
28°	3.69	5.19	6.31	9.09	13.30

2. Of the *planted* bulbs the tables 4 and 4*a* give the average length of the outer leaf above the ground at the point of time, that the leaves of the greater part of the bulbs already showed above ground.

The measuring was rather inaccurate, especially on account of the rough surface of the soil, so that only the great differences are of value.

TABLE 4. *l'Innocence.*

Average lengths of the outer foliage-leaves above ground in mms per bulb after removal of the cover (peat-litter) on January 28, 1927 (× = under the surface or on the same level).

		3 weeks		5 weeks		$7^1/_2$ weeks		12 weeks
Preliminary exposure	17°	15.5	—	17.3	—	—	—	3.0 and 5.8
	20°	16.0	×	13.3	1.3	4.3	6.0	2.8
	23°	27.8	20.8	26.0	7.3	10.5	4.0	2.0
	25.5°	**59.0**	28.8	25.5	21.3	6.0	4.5	2.3
	28°	36.3	25.0	31.0	26.8	22.5	23.5	6.0
After-exposure to:		13°	17°	13°	17°	13°	17°	—

TABLE 4*a*. *la Victoire*, as table 4.

		3 weeks		5 weeks		$7^1/_2$ weeks		12 weeks
Preliminary exposure	17°	4.0	—	3.8	—	1.5	—	×
	20	10.3	2.8	12.8	×	2.3	×	×
	23°	**31.3**	1.3	13.0	1.8	6.3	×	×
	25.5°	**23.8**	1.5	19.0	0.8	6.5	5.3	×
	28°	**31.8**	2.8	19.3	5.3	7.8	5.0	×
After-exposure to:		13°	17°	13°	17°	13°	17°	—

From the tables 4 and 4*a* this conclusion may be drawn: for the stretching of the leaves up to this time the after-treatment in 13° is better than in 17°. Further it may be stated, that l'Innocence 3 weeks 25.5° and next 13° surpasses the other treatments favourably. Here too it appears, that the optimum which is found after 12 weeks 17° with the leaflets in embryonic condition (table 3) is shifted when reckoned over a longer period (during the stretching).

For la Victoire (table 4*a*) we find a corresponding optimum in January in 3 weeks 23°, 25.5° and 28° with after-treatment in 13°.

Number of sheath-leaves in l'Innocence.

On July 5 each of the 10 bulbs examined had 2 sheath-leaves. In the bulbs examined after exposure (fixed material) there occurred in each

TABLE 5. *l'Innocence.*

Number of sheath-leaves per 10 bulbs. (The number at the beginning of the experiment was 20 per 10 bulbs.

	After 3 weeks	5 weeks	7½ weeks	12 weeks
17°	19	17	19	16
20°	17	19	19	18
23°	19	19	19	16
25.5°	17	19	16	18
28°	19	17	17	17

group of 10 at least 1 bulb with one sheath-leaf (table 5). Of the 200 bulbs 155 bulbs had two sheath-leaves, 44 bulbs one sheath-leaf and 1 bulb three sheath-leaves (this latter in 3 weeks 17°). That a certain exposure can influence this figure cannot be concluded from this.

§ 3. *The direct effect of 17°—28° during 3 to 12 weeks on the development of the floral whorls in l'Innocence.*

About the 210 fixed bulbs the results as to number of foliage-leaves and sheath-leaves and length of the outmost foliage-leaf have already been communicated in § 2. In the present § the other observations on these bulbs are mentioned, viz. in what stage of development the vegetation-point that is to yield the flower-cluster for the following spring, was found in consequence of the 20 temperature-exposures.

In order to enable us to represent the progress of the formation of the floral whorls BLAAUW (1920 and 1924) adopted 10 stages of development. Stage I is still a simple vegetation-point, in stage X this has grown into a cluster, the lowest flowers of which are quite complete. In the variety l'Innocence the stages can in the main be distinguished in the same way, as in the previously examined Queen of the Blues. What differences there are, will be discussed afterwards.

The stages of development in question have been drawn from nature and lithographed by Mr. VAN TONGEREN. The objects, stained with a strong aqueous solution of iodine and iodide of potassium were examined (also during the drawing) through a binocular microscope. The magnification has been given with the illustrations (45—50 ×). What has been drawn, was partly selected from the material of this research, partly from other l'Innocence-material of equal age. In determining the stages the lowest flowers were examined as these are the first to develop. The number of flowers, their size and shape do not act a part in this. Neither does the origin and development of the new vegetation-point, that (normally) will pass on to flower-formation the next summer.

In the plates denotes: VP, vegetation-point; L, foliage-leaf; LL, scar foliage-leaf; L*, rudimentary foliage-leaf; NVP, new vegetation-point; BR, bract; BLP, flower-primordium; NPH 1, first leaf of the new phyllome-series; S, bractlet; T I and T II, tepals of the outer and the inner whorl; M I and M II stamens of the outer and the inner whorl; VD, carpel. When of the NVP and of the first sheath-leaf of the NVP the swelling is not yet visible (not any or extremely little external differentiation), but the spot where these organs will arise, is already visible (of the NVP through strong staining by the aqueous solution of iodine and iodide of potassium), this was designated (NVP) and (NPH 1).

The subsequent stages in l'Innocence are:

I. The vegetation-point VP is still splitting off leaves (L) and is still low (fig. 1).

II. The vegetation-point has finished splitting off leaves and is raised, while no differentiation is visible but a weak indication (NVP) or a slight swelling NVP of the new vegetation-point (figs. 2, 3 and 4).

III. Besides the new vegetation-point which gets clearer now some crescentshaped prominences BR are visible (figs. 5 and 6).

IV. In the lower primordia we now see the bract (as a roundish prominence) and the primordium of the proper flower BLP (likewise as a roundish prominence) separated from each other by a furrow (figs. 7 and 8).

V. The three outer tepals T I are to be distinguished as three independent primordia (fig. 9).

VI. The three inner tepals T II id. (figs. 10 and 11).

VII. The three outer stamens M I id.

VIII. The three inner stamens M II id. (fig. 12).

IX. The three carpels VD id. (fig. 13).

X. The three carpels are raised, while the margins of each carpel are turned in (fig. 14).

Besides the above description the plates and stages need the following explanation of a more secondary nature:

Figures 1 and 2 have been drawn in the same magnification and also in the same position, so that the height of the two vegetation-points (fig. 1 and fig. 2) are comparable in the drawings. Figs. 3 and 4 give the growing-point seen from above.

Figure 4 has been drawn from the same object as fig. 3, but turned about 180 degrees.

In fig. 3 the last fully formed foliage-leaf has been slightly lifted to show how this leaf covers the vegetation-point. This innermost foliage-leaf namely frequently presses upon the growing-point or the developing flower-cluster so heavily, that a distinct ridge or dent is visible on it. This therefore has no actual signification with respect to the formation of organs; this should be borne in mind, because this may give rise to errors in judging the object. In figures 2, 3, 4, 5 and 10 the dent thus originated has been marked with a ×.

The leaflet last formed (L* in figures 2 and 5) has often not even been finished and is arrested as a rudimentary organ in the shape of a scale. This rudimentary leaflet may be followed as far as stage X and further, but it is no more visible in the illustrations after stage III.

In stage IV— (which approaches stage IV very closely) we see (fig. 7) the first indication of a sheath-leaf in the new vegetation-point; in the figure (NPH 1). In the following fully drawn clusters the further development may be traced.

In stage IV (fig. 8) in the lowest flowers we see the first differentiation of a bractlet, indicated with S, the further development of which may likewise be traced in some flowers in figures 9 and 10. See on this bractlet: BLAAUW 1920, Summary § 3.

In stage VI the entire flower-cluster has been given (fig. 10), but the first (lowest) flower, indicated BLP 1 of this same flower-cluster has been opened and drawn in the right hand top corner of the plate (fig. 11), in order to indicate the feature of stage VI, i.e. the differentiated condition of the inner tepals. Of the following stages but one of the flowers has been drawn after being opened. This latter is necessary, because after stage V the tepals begin to overlap the further inward parts of the flower. This opening sometimes causes the turned down tepals to be torn loose at the inside of the base, which is shown bij the scars in the drawings.

The differences with Queen of the Blues with regard to the stages.

Ad stage II. In Queen of the Blues stage II was considered the condition in which the vegetation-point is raised, whilst no further differentiation is visible. In l'Innocence it was observed, that the new vegetation-point may already be visible in the axil of the last leaflet (by cell-divisions at that place or by some swelling) fully split off (binocular; staining with iodine and iodide of potassium), when the vegetation-point has finished splitting off leaves and before there is any indication of a bract.

Characteristic of stage II is only the rise of the vegetation-point, not the appearance of the new vegetation-point, because also in judging the later stages we have left this out of account.

Ad stages III and IV. In Queen of the Blues stage III has been called the condition, in which the 1st or the 1st and 2nd flower-primordia are visible as weak swellings. Besides the new growing-point at the base is not mentioned before this.

In l'Innocence we first observed a crescent-shaped roundish prominence of the bract (= stage III), while the part above this which is to give rise to the proper flower-primordium does not show any prominence. Not until after this there appears above the bract a prominence of the proper flower. At first this is lower than the bract (= III+ or IV—), but it will soon be higher (= IV). Further the bract lags considerably behind in growth.

In limiting the stages we can also mention as a criterion for stage III the rise of a single crescent-shaped prominence. Then it is left undecided for the present, whether this prominence is the bract or a primordium which will yet be differentiated into a bract and a flower.

Ad stage VII. For Queen of the Blues (and also for l'Innocence) stage VII is reached, when the three outer stamens are visible as independent organs. But by that time in the examined material of l'Innocence there was always something to be seen of the inner whorl; for that reason the stage is called VII+ or VIII—. Now it is possible, that not all the stages and transitions from one stage to another could be observed in this material, the number of different treatments being limited here. Yet the impression remains that the stages VII and VIII, when they do not fairly coincide, succeed very quickly, i.e. in this way, that the inner whorl commences forming, when the outer is not yet finished.

The progress of the flower-cluster-formation.

This has been synoptically represented in a "curve of dots" (fig. 15). (See Blaauw 1924 and Luyten, Joustra, Blaauw 1925). Each dot indicates a bulb and is placed with the stage of development (ordinate), in which the vegetation-point or the lowest flower were, while the treatment is indicated on the abscis. A +stage was put in the upper and a —stage in the lower half of a square. In the advanced stages IX the anthers could be measured under the binocular; their length was ca 0.2 mm. As smallest measure in stage X ca. 2.5 mms. was found. A sub-division of stage X or rather the development after stage IX has been rendered according to the length of the anthers. It is necessary therefore to take into account, that on the ordinate above and below the heavy line values have been plotted which were obtained in different ways.

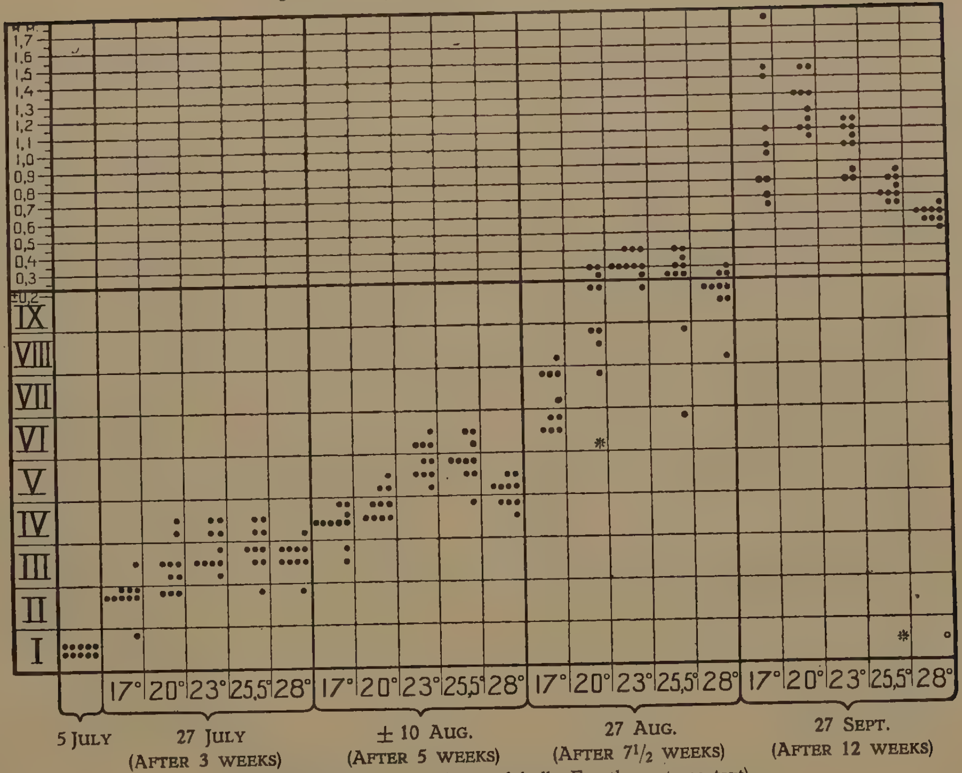
Here a discussion follows of the Fig. 15 in question:

1. *On July 5* at the beginning of the experiment all of the 10 examined bulbs were in stage I. It was however the end of the period of splitting off leaves as appears from tables 1 and 2.

2. *After 3 weeks* 25.5° has progressed farthest, though 23° and 28° are falling behind but little.

3. *After 5 weeks* the flower-cluster is further developed than in whatever temperature after 3 weeks. 23° and 25.5° are farthest. It was

Fig. 15. The progressing flower-cluster-formation.



(* = diseased bulb, o = abnormal bulb. For the rest see text).

remarkable in this case, that in 25.5° in two bulbs the new vegetation-point at the foot of the cluster had already passed on to flower-formation. In both bulbs this had already attained stage IV.

4. *After 7½ weeks* 23° and 25.5° are farthest and besides, especially in 23° very uniformly developed. The development of the 10 bulbs in 20° is particularly unequal. One of the bulbs in 20° was diseased, but reached stage VI after all (indicated with * in fig. 15).

5. *After 12 weeks* 17° is farthest, but (just as in Queen of the Blues) most unequal. In this case anthers were measured of a length of 0.65 to 1.75 mms. Of a failure of the flower-cluster, as in Queen of the Blues, there is no question in l'Innocence in 17°. But the Queen of the Blues examined at the time was a year younger with a circumference of 7.5—9 cms and accordingly already therefore possessing slighter flowering-capacities. Averagely most advanced is 20°. Most uniform in development are the bulbs in 28° (anthers of a length of 0.5 to 0.65 mm) and in 25.5° (from 0.65 to 0.85 mm.) ; 25.5° is the farthest of these. In 25.5° there was a diseased bulb, which had not progressed at all in its development. In 28° there was a similar bulb likewise entirely lagging behind in development, which was abnormal in a different way. Here too in 25.5° the new vegetation-point in one of the bulbs had proceeded to flower-formation (stage V+). This was not the bulb that had not developed at all.

§ 4. *The total effect on the flower-cluster in l'Innocence and la Victoire.*

In order to trace the total effect of the various exposures the bulbs were planted out after 12 weeks (on Sept. 27, 1926) in a cistern with fixed groundwaterlevel of 60 cms. (described in Blaauw 1922). The total effect on number and length of the leaves has already been rendered in § 2 (together with the direct effect) ; in this § the total effect on the flower-clusters follows. See the tables 6 and 6a. At the top (between the double and the heavy horizontal lines) the exposure has been given. Next the number of planted bulbs follows. 20 bulbs were taken per treatment, but in some cases, where the figure 19 is found, one has dropped out. Next there is added to the dates the number of flower-clusters, which started flowering on these dates. Then the number of flowering and non-flowering bulbs and the average number of flowers per cluster ; at the foot of these the mean error of this average. The signification of the * we find at the close of this §. Mind, that in table 6 two groups are found with an exposure of 12 weeks 17°.

Beginning of flowering.

On March 17, 1927 the first flowers of la Victoire and on March 19 those of l'Innocence came into bloom. The number of clusters which came

TABLE 6. Flowering of *l'Innocence* (see § 4).

For........	3 weeks								
in..........	17°	20°		23°		25.5°		28°	
next in......	13°	13°	17°	13°	17°	13°	17°	13°	17°
Number of planted bulbs	20	20	20	20	20	20	19	20	20
Beginning of flowering on: 19 Mrch						1	1		
21 „		1		13	8	10	9	10	16
23 „	5	5	4	6	10	2	8	8	3
25 „	2	1	12		2		1		
27 „			2						
29 „			1						
31 „									
2 April									
4 „			1						
Number of flowering bulbs (= n)	7	7	20	19	20	13	19	18	19
Number of non-flowering bulbs	13	13		1		7		2	1
Number of flowers per cluster (= M)	9.3*	8.6*	10.1	9.9*	9.7(*)	**10.7***	9.1	8.2*	10.5
$m = \pm$	0.9	0.9	0.3	0.5	0.5	0.4	0.5	0.4	0.4

For........	5 weeks								
in..........	17°	20°		23°		25.5°		28°	
next in......	13°	13°	17°	13°	17°	13°	17°	13°	17°
Number of planted bulbs	19	20	20	19	20	20	20	20	20
Beginning of flowering on: 19 Mrch				$7_{(8)}$		$7_{(11)}$		6	3
21 „	3	9		12	9	13	19	14	15
23 „	3	5	3		10		1		
25 „	3	3	10		1				1
27 „									
29 „			6						
31 „									
2 April									
4 „									
Number of flowering bulbs (= n)	9	17	19	19	20	20	20	20	19
Number of non-flowering bulbs	10	3	1						1
Number of flowers per cluster (= M)	10.6*	11.4*	10.4*	10.6*	10.8*	**11.6***	**11.5**	11.2*	9.7
$m = \pm$	0.6	0.4	0.4	0.5	0.5	0.4	0.4	0.2	0.3

For........	7½ weeks								
in..........	17°	20°		23°		25.5°		28°	
next in......	17°	13°	17°	13°	17°	13°	17°	13°	17°
Number of planted bulbs	20	19	20	20	20	20	20	20	20
Beginning of flowering on: 19 Mrch				1					
21 „		2		10	3	9	5	16	6
23 „	2	10	8	8	16	10	15	3	11
25 „	4	6	8	1				1	3
27 „	6	1	2		1				
29 „	5		2						
31 „	3								
2 April	.								
4 „									
Number of flowering bulbs (= n)	20	19	20	20	20	19	20	20	20
Number of non-flowering bulbs						1			
Number of flowers per cluster (= M)	10.0*	10.4*	10.0*	11.1*	11.0	**11.3**	**11.9**	10.9*	10.1
$m = \pm$	0.5	0.6	0.5	0.4	0.3	0.3	0.6	0.5	0.5

For........	12 weeks				
in..........	17°	20°	23°	25.5°	28°
next in......	—	—	—	—	—
Number of planted bulbs	20	20	20	20	20
Beginning of flowering on: 19 Mrch					
21 „					
23 „	1				
25 „	4	9	6		
27 „	9	7	11	5	
29 „	3	4	2	12	10
31 „			1	1	4
2 April	1			2	4
4 „					2
Number of flowering bulbs (= n)	18	20	20	20	20
Number of non-flowering bulbs	2				
Number of flowers per cluster (= M)	9.5*	10.1*	10.9*	**11.2***	11.1*
$m = \pm$	0.4	0.4	0.3	0.4	0.4

TABLE 6a. Flowering of *la Victoire* (see § 4).

For.........	3 weeks									5 weeks									7½ weeks									12 weeks				
in..........	17°	20°		23°		25.5°		28°		17°	20°		23°		25.5°		28°		17°	20°		23°		25.5°		28°		17°	20°	23°	25.5°	28°
next in......	13°	13°	17°	13°	17°	13°	17°	13°	17°	13°	13°	17°	13°	17°	13°	17°	13°	17°	13°	13	17°	13°	17°	13°	17°	13°	17°	—	—	—	—	—
Number of planted bulbs	20	19	20	20	19	20	20	20	20	19	19	19	20	20	20	19	20	20	20	20	20	19	19	20	20	20	20	20	20	19	20	20
Beginning of flowering on:																																
17 Mrch				2		3							1		6		1									1						
19 „		3		7		5		2			3		13		14		14	6				11		3		6						
21 „	3	5		8	5	9	7	11	5	4	11		6	12		15	5	11	3	13		5	2	17	16	12	13					
23 „	1	7	2	2	6	3	10	5	11	3				6		4		2	9	5	4	3	11		4	1	5					
25 „	1	2	7		5		3		2	6	4	7		2				1	5	2	11		6				2					
27 „	3	1	4		3			2	2	5	1	11							2		5							3	2			
29 „	6		7									1																13	16	10	12	16
31 „	1																											4	2	6	6	4
2 April																														2	1	
4 „																														1	1	
Number of flowering bulbs (= n)	15	18	20	19	19	20	20	20	20	18	19	19	20	20	20	19	20	20	19	20	20	19	19	20	20	20	20	20	20	19	20	20
Number of non-flowering bulbs	5	1		1						1									1													
Number of flowers per cluster = M	14.9*	19.3*	24.6	19.4*	23.8	18.3*	20.4	13.9*	19.3	21.3*	23.4*	24.4	22.3*	23.7	20.5*	20.3	19.5	18.1	24.9(*)	27.0	24.2	21.8	22.1	20.0	22.0	19.4	17.8	23.9	25.4	24.1	20.8	19.5
$m = \pm$	1.5	1.1	0.9	1.1	1.2	0.5	1.1	0.8	0.5	1.5	2.1	0.7	0.8	1.0	0.6	0.6	1.4	0.7	0.7	1.3	0.9	0.7	0.7	0.6	0.9	0.6	0.5	1.1	1.8	1.1	0.7	0.9

into bloom on these days (and further every other day) have been given in the tables.

l'Innocence flowers earliest in an exposure of 5 weeks to 25.5°, 23° and 28° with an after-treatment to 13°. In 23° and 25.5° the figures 8 and 11 are placed between brackets. We doubted namely in these cases whether we should note 7 or 8 clusters for each in 23° and 11 clusters in 25.5°, but at last we decided on 7 for each. Yet it may appear from this, that 25.5° is a little in advance.

La Victoire has a conspicuous start in an exposure of 5 weeks to 25.5° with an after-exposure to 13°.

Finally it is evident that for a rapid flowering the after-exposure to 13° is more favourable than to 17°.

When we compare the celerrimum for *flower-formation* in fig. 15 with the rate of *coming into bloom* in table 6, we see parallel "curves". After 3 and 5 weeks and 7½ weeks 23° and 25.5° and also 28° are the first to flower in each case, while the flower-cluster-formation is likewise most advanced in these exposures. After an exposure of 12 weeks to the same temperature the flowers from 17° are the first to bloom, from 28° the last. The rate of flower-formation in these temperatures is parallel. Throughout the winter this celerrimum was maintained.

Accordingly a celerrimal effect is obtained by exposure first for some weeks to a high temperature (25.5°, 23° to 28°), next to a low temp. (17°, still better: 13°). If however in the later weeks the temperature is kept high, this retards so much, that permanently 17° and 20° are more rapid than permanently 23°—28°.

It is a striking fact, that this effect in these early varieties is achieved in the same temperatures as in the late variety Queen of the Blues. Especially with a view to treatment for early flowering it was important to control this point once more in well-known early varieties.

Number of flowering and non-flowering bulbs.

It has appeared from fig. 15, that in l'Innocence every growing-point has proceeded to flower-formation in the chosen temperatures. From table 6 it is to be seen, that it may happen, that the flower does not develop in the lower temperatures, especially in an after-treatment to 13°, preceded by too short a preliminary exposure to but 20° or 17°. After 5 weeks to and above 23° nearly every flower-cluster succeeds, both in the after-exposure to 13° and to 17°.

In la Victoire (table 6*a*) fewer bulbs have fallen behind in the chosen temperatures, from which it may be concluded, that this variety can endure a lower temperature.

The Queen of the Blues previously examined was a year younger and already for that reason the number of flowering bulbs is smaller (see § 6).

Number of flowers per cluster.

The number of flower-clusters (= n) varies from 7 to 20 in the two varieties; it usually amounts to 19 or 20 (tabs. 6 and 6*a*). The mean error of the average number of flowers per cluster was calculated according to JOHANNSEN with the formulas:

$$m = \pm \frac{\sigma}{\sqrt{n}} \text{ and } \sigma = \pm \sqrt{\frac{\Sigma\, p\, a^2}{n} - b^2}.$$

For l'Innocence the mean error is fairly uniformly low. In 3 weeks 17° and 20° and next in 13° it is greatest (number of flower-clusters = 7). For la Victoire the mean errors differ more and are greater on account of the deviations of the variants of the mean value being greater and more irregular, owing to the great number of flowers that occurs on the clusters in this variety.

In what temperature must the preliminary exposure take place and after what time? Taking the mean error (M ± 3 m) into consideration, we can but point out differences between the greatest and the smallest numbers in l'Innocence. The ever recurring optimum at ca 25.5° after 3, 5, ca 8 and 12 weeks is however striking. Even clearer is a similar optimum in la Victoire, but here it is found in 20° (after 3 and 5 weeks also in 23°). In 23°, 25.5° and 28° the numbers get smaller and smaller. The preliminary treatment must last longer in the two varieties than 3 weeks, though this is not so necessary, when the above-mentioned optima are applied.

If we consider whether the after-treatment to 17° or to 13° is necessary, we must not give preference to either of them in l'Innocence. For only where the slightest number of flowers occurs (3 w. 28° + 9 w. 13°) there is, taking the mean error into account, a real difference to be stated between the treatment in 17° and in 13°. In la Victoire this is different: When the preliminary exposure to 20°—28° lasts but 3 weeks, 17° is better, because in that case a greater number of flowers remains rudimentary in 13°. When the preliminary exposure to 20°—28° lasts 5 weeks, the number of rudimentary flowers is smaller and when an after-exposure to 13° or to 17° follows no real difference in number can be pointed out. Accordingly here (with the mean error) it may be more or less traced in the table, where the number of rudimentary flowers is great.

Where these rudimentary flowers occur in la Victoire the following indications are made: In the tables 6 and 6*a* sub "number of flowers per cluster" an asterisk or an asterisk between brackets has been placed. In the case of la Victoire this means, that in those cases the clusters bore a number of apical flowers (from 1 to 5) which showed aberrations due to the treatment (as in Queen of the Blues, BLAAUW 1924, first part § 8). With (*) is indicated, that it was only distinctly observed on one of the clusters. Only those flowers were taken into consideration, which could be clearly distinguished with the naked eye. It occurs in 3 and 5 weeks only then, when an after-treatment to 13° took place. In 5 weeks 28° and next 13° it does no more occur; it was observed still however at a cluster in 7½ weeks 17° and next 13°.

Especially when many apical flowers fall behind, this goes together with a thinner common flower-stalk and smaller and paler flowers.

Though therefore in the after-treatment to 13° the celerrima are found, they are no optima here in the sense that the flower-cluster is best developed and richest in flowers.

In the case of l'Innocence * and (*) indicate, that the tepals in those cases were slightly greenish at the apex. Generally this does not occur in the after-treatment to 17°, but it does occur when the bulbs have been kept in the same temperature for 12 weeks.

§ 5. *Increase in weight of the bulbs.*

On Sept. 27, 1926 (after the treatment in the various temperatures) the sets of bulbs were weighed and next planted in a cistern with equal soil and ground-water-level, as has already been described in § 4. From this date to the date of lifting (July 4, 1927) they have been, though all in an equal measure, exposed to various influences of the weather. The differences, we find in the tables 7 and 7*a* therefore are due to the different exposures in the previous summer, while after that for months together the climate has influenced them all equally.

The small figures in the tables denote the weight per bulb (an average of 20 bulbs) on Sept. 27, 1926 and on July 5, 1927 (for l'Innocence) and on July 6, 1927 (for la Victoire). So after lifting they were weighed on two subsequent days, which has been recorded, because the loss of weight in one day is already worth mentioning. After each brace the difference, i.e. the average increase in weight, has been given in large figures.

TABLE 7. *l'Innocence.*

Increase in weight per bulb in grams from Sept. 27, 1926 (date of planting) to July 4, 1927 (date of lifting)

		3 weeks		5 weeks		7½ weeks		12 weeks
Preliminary exposure	17°	$\left.\begin{matrix}47.1\\28.4\end{matrix}\right\}$ 19.7	—	$\left.\begin{matrix}48.1\\28.5\end{matrix}\right\}$ 19.6	—	—	—	$\left.\begin{matrix}52.6\\27.7\end{matrix}\right\}$ 24.9 and $\left.\begin{matrix}50.3\\27.7\end{matrix}\right\}$ 22.6
	20°	$\left.\begin{matrix}51.8\\28\ 1\end{matrix}\right\}$ 23.7	$\left.\begin{matrix}54.5\\27.7\end{matrix}\right\}$ 27.8	$\left.\begin{matrix}50.4\\28.4\end{matrix}\right\}$ 22.0	$\left.\begin{matrix}50.0\\27.4\end{matrix}\right\}$ 22.6	$\left.\begin{matrix}50.1\\27\ 6\end{matrix}\right\}$ 22.5	$\left.\begin{matrix}49.8\\27.7\end{matrix}\right\}$ 22.1	$\left.\begin{matrix}51\ 9\\27.9\end{matrix}\right\}$ 24.0
	23°	$\left.\begin{matrix}49.4\\28.3\end{matrix}\right\}$ 21.1	$\left.\begin{matrix}54.2\\28.2\end{matrix}\right\}$ 26 0	$\left.\begin{matrix}49.7\\28.1\end{matrix}\right\}$ 21.6	$\left.\begin{matrix}51.7\\28.2\end{matrix}\right\}$ 23.5	$\left.\begin{matrix}52.0\\28.3\end{matrix}\right\}$ 23.7	$\left.\begin{matrix}54.7\\27.8\end{matrix}\right\}$ 26.9	$\left.\begin{matrix}55.1\\28.6\end{matrix}\right\}$ 26.5
	25.5°	$\left.\begin{matrix}46.9\\28.5\end{matrix}\right\}$ 18.4	$\left.\begin{matrix}54.1\\27.4\end{matrix}\right\}$ 26.7	$\left.\begin{matrix}51.6\\28.2\end{matrix}\right\}$ 23.4	$\left.\begin{matrix}58\ 0\\28.0\end{matrix}\right\}$ **30.0**	$\left.\begin{matrix}55.3\\27.9\end{matrix}\right\}$ 27.4	$\left.\begin{matrix}53.6\\28.0\end{matrix}\right\}$ 25.6	$\left.\begin{matrix}53.7\\28.4\end{matrix}\right\}$ 25.3
	28°	$\left.\begin{matrix}57.9\\27.7\end{matrix}\right\}$ **30.2**	$\left.\begin{matrix}58.3\\27.7\end{matrix}\right\}$ **30.6**	$\left.\begin{matrix}52.3\\28.1\end{matrix}\right\}$ 24.2	$\left.\begin{matrix}56.3\\28.0\end{matrix}\right\}$ 238.	$\left.\begin{matrix}53\ 5\\27.8\end{matrix}\right\}$ 25.7	$\left.\begin{matrix}54.3\\27\ 9\end{matrix}\right\}$ 26.4	$\left.\begin{matrix}56.4\\28.6\end{matrix}\right\}$ 27.8
After-exposure to:		13°	17°	13°	17°	13°	17°	—

TABLE 7a. *la Victoire*, as table 7.

		3 weeks		5 weeks		7½ weeks		12 weeks
Preliminary exposure	17°	$\left.\begin{matrix}37.1\\25.5\end{matrix}\right\}$ 11.6	—	$\left.\begin{matrix}40.8\\25.7\end{matrix}\right\}$ 15.1	—	$\left.\begin{matrix}42.4\\25.7\end{matrix}\right\}$ 16.7	—	$\left.\begin{matrix}46.0\\25.8\end{matrix}\right\}$ 20.2
	20°	$\left.\begin{matrix}43.3\\25.4\end{matrix}\right\}$ 17.9	$\left.\begin{matrix}45.6\\25.6\end{matrix}\right\}$ 20.0	$\left.\begin{matrix}45.2\\25.4\end{matrix}\right\}$ 19.8	$\left.\begin{matrix}46.0\\25.4\end{matrix}\right\}$ 20.6	$\left.\begin{matrix}41.7\\25.4\end{matrix}\right\}$ 16.3	$\left.\begin{matrix}45.0\\25.8\end{matrix}\right\}$ 19.2	$\left.\begin{matrix}46.1\\25.6\end{matrix}\right\}$ 20.5
	23°	$\left.\begin{matrix}43.7\\25.4\end{matrix}\right\}$ 18.3	$\left.\begin{matrix}46.7\\25.6\end{matrix}\right\}$ 21.1	$\left.\begin{matrix}44.6\\25.2\end{matrix}\right\}$ 19.4	$\left.\begin{matrix}44.1\\25.4\end{matrix}\right\}$ 18.7	$\left.\begin{matrix}46.4\\25.4\end{matrix}\right\}$ 21.0	$\left.\begin{matrix}45.9\\25.8\end{matrix}\right\}$ 20.4	$\left.\begin{matrix}46.5\\26.2\end{matrix}\right\}$ 20.3
	25.5°	$\left.\begin{matrix}44.6\\25.2\end{matrix}\right\}$ 19.4	$\left.\begin{matrix}47.2\\25.5\end{matrix}\right\}$ 21.7	$\left.\begin{matrix}47.4\\25.2\end{matrix}\right\}$ 22.2	$\left.\begin{matrix}51.3\\25.5\end{matrix}\right\}$ 25.8	$\left.\begin{matrix}49.3\\25.3\end{matrix}\right\}$ 24.0	$\left.\begin{matrix}48.3\\25.5\end{matrix}\right\}$ 22.8	$\left.\begin{matrix}49.1\\26.4\end{matrix}\right\}$ 22.7
	28°	$\left.\begin{matrix}46.8\\25.0\end{matrix}\right\}$ 21.8	$\left.\begin{matrix}48.5\\25.4\end{matrix}\right\}$ 23.1	$\left.\begin{matrix}46.5\\25.1\end{matrix}\right\}$ 21.4	$\left.\begin{matrix}48.7\\25.1\end{matrix}\right\}$ 23.6	$\left.\begin{matrix}45.8\\25.1\end{matrix}\right\}$ 20.7	$\left.\begin{matrix}48.4\\25.3\end{matrix}\right\}$ 23.1	$\left.\begin{matrix}49.5\\26.2\end{matrix}\right\}$ 23.3
After-exposure to:		13°	17°	13°	17°	13°	17°	—

BLAAUW (1924) points out, that little attention must be paid to slight differences in weight. In this case this is evident from the 2 sets of 20 bulbs which (one by mistake) were both left in 17° for 12 weeks. Here there is a difference of 2.3 grms, so that therefore no value must be attached to 10 % increase in weight.

On comparing table 7 with 7*a*, it strikes us at once, that the increase in weight in l'Innocence is in every case greater than in la Victoire. On our starting the experiment the bulbs were equally large for the two varieties; positively the more vigorous leaves of l'Innocence with larger assimilating surface than those of la Victoire act an important part here.

Nearly everywhere the after-treatment in 17° appears to be more favourable than in 13°, especially in l'Innocence. An exception to this is for instance 7½ weeks 25.5° + 4½ w. 13°; yet this difference is smaller than 10 %, so that it is a question whether it must be attached any value to. But it is pointed out here, because in Queen of the Blues (BLAAUW 1924, tab. 32) there was also stated greater increase in weight in this case.

Taking the preliminary exposures also into consideration the following exposures are optimal for l'Innocence: 5 w. 25.5° + 7 w. 17° and 3 w. 28° + 9 w. 13° or 17°. For la Victoire 5 w. 25.5° + 7 w. 17°. For Queen of the Blues (BLAAUW 1924, tab. 8) there is an optimum at *eight* weeks 25.5° + 7 w. 17°. In these early varieties therefore optima are found in the higher temperatures (25.5° and 28°), during a shorter period (3 and 5 weeks).

§ 6. *Summary in connection with the application. Comparison with Queen of the Blues.*

With respect to the growing out of all leaves in the field in spring it has appeared, that the higher the temperature of the preliminary exposure, the shorter it should last. For l'Innocence (tab. 2) this temperature may be lower, and needs last shorter than for Queen of the Blues (BLAAUW 1924, tabs. 2 and 24). For la Victoire (tab. 2*a*) this temperature may be lower yet and the preliminary exposure shorter than for l'Innocence. As to the combination of optima (BLAAUW 1924, §§ 1 and 9) this causes no difficulties, as all following optima for this are found as a rule at so high a temperature and for so long a period, that all leaves shoot.

For the stretching of the foliage till January (judged from the outmost foliage-leaf) for l'Innocence (tab. 4) a treatment in 25.5° for 3 weeks and next 13° is optimal. For la Victoire (table 4*a*) this may also be 23° and 28°. The after-exposure to 17° is not favourable in this case, especially not for la Victoire. For Queen of the Blues (BLAAUW 1924, tab. 4 and § 11) this lies in a higher temperature, which is to be applied for a longer period.

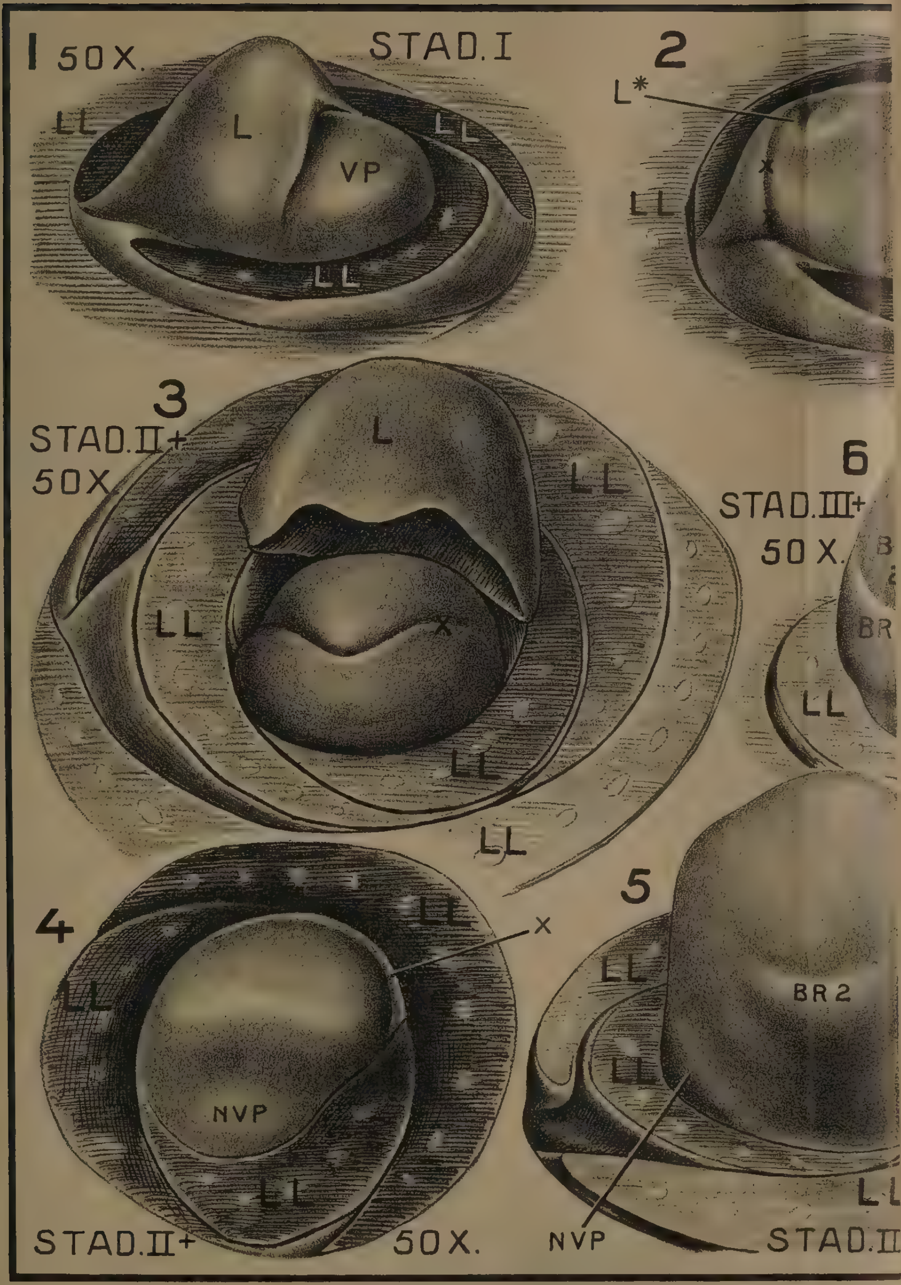
For celerrimal flowering in l'Innocence and la Victoire 25.5° is necessary for 5 weeks and then 13° till the planting-period (tabs. 6 and 6*a*). In l'Innocence the lower surfaces of the tips of the tepals were slightly green in this case, which does not do much harm to the beautiful appearance of the flower. In la Victoire however the apical flowers had remained rudimentary in 13°. This however was stated after a stay for months together outdoors in the field. For a celerrimal flowering (forcing) indoors the results may be more favourable and at any rate an after-treatment in 17° would cause too great a delay i.e. no celerrimal effect. In Queen of the Blues (according to experiments not yet published) the celerrimum of flowering is also found at 25.5° or 28° for 5 weeks, followed by an after-treatment in 13°.

Non-flowering bulbs (tabs. 6 and 6*a*) occur in lower temperatures than the above-mentioned optima, so that — with a bulb-circumference of 12 to 13 cms — we need not take this into account for the combination of the optima.

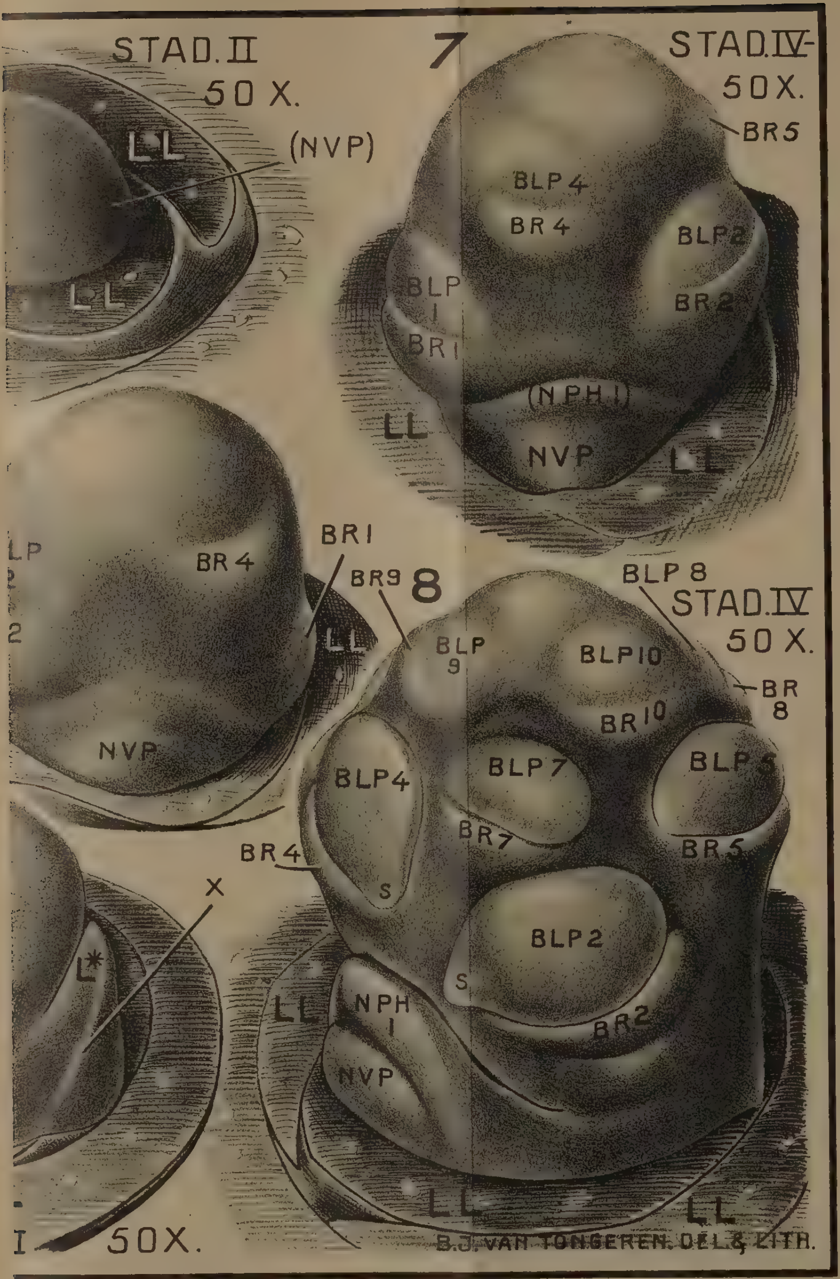
Queen of the Blues of 7.5—9 cms (BLAAUW 1924, tab. 14) is much closer to the limit of flowering-ability ; those of 12—13 cms (ibid. tab. 23) are perfectly parallel, as far as the exposure was identical, with these early varieties. The Queens of 7.5—9 cms must not be compared in this respect with the older bulbs of the early varieties, because these have a more vigorous flowering-capacity and consequently yield a greater number of flowering bulbs.

Of la Victoire and l'Innocence it may be said, that with a short preliminary exposure in this respect the former can endure a lower temperature better than l'Innocence.

With respect to the greatest number of flowers per cluster for l'Innocence (tab. 6) ca 25.5° for 5 weeks or 7½ weeks and next 13° or 17° is optimal. In the same way the Queen of the Blues of



HYACINTHUS ORIENTA



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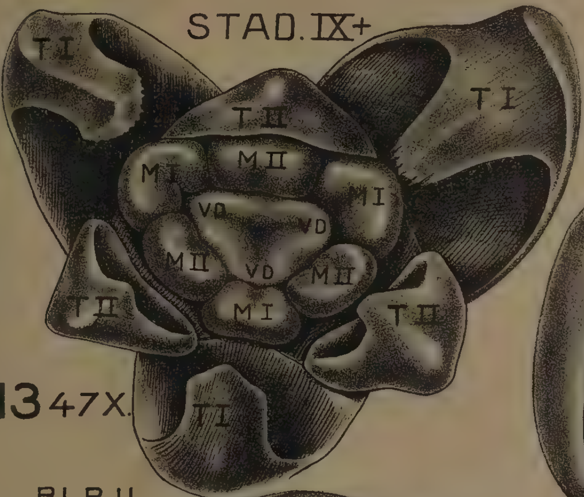
DRUK v. P.J. MULDER & Zn LEIDEN (HOLLAND)

ALIS. (L'INNOCENCE)



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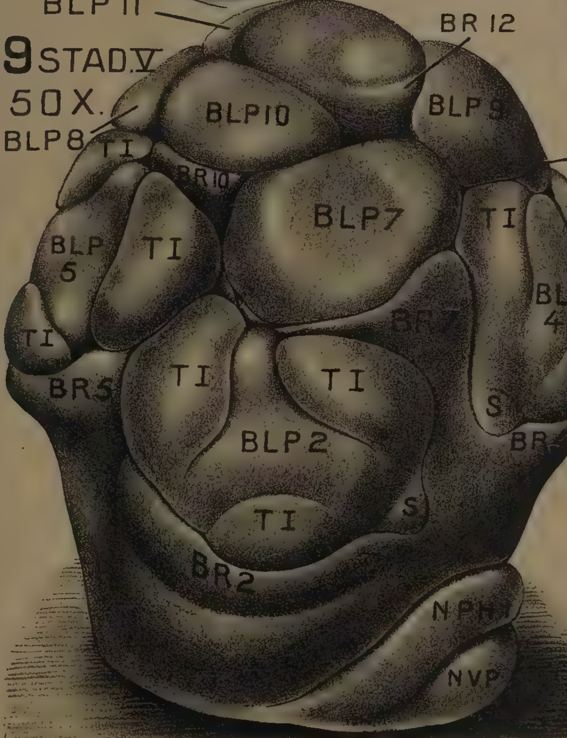

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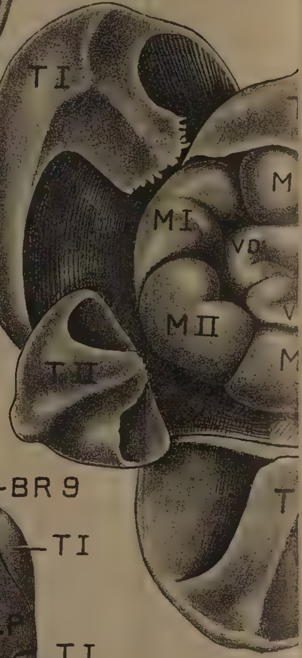

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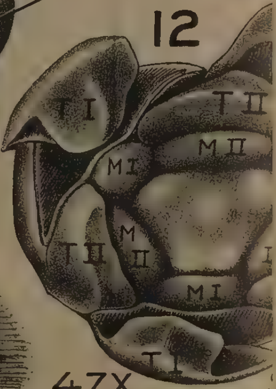

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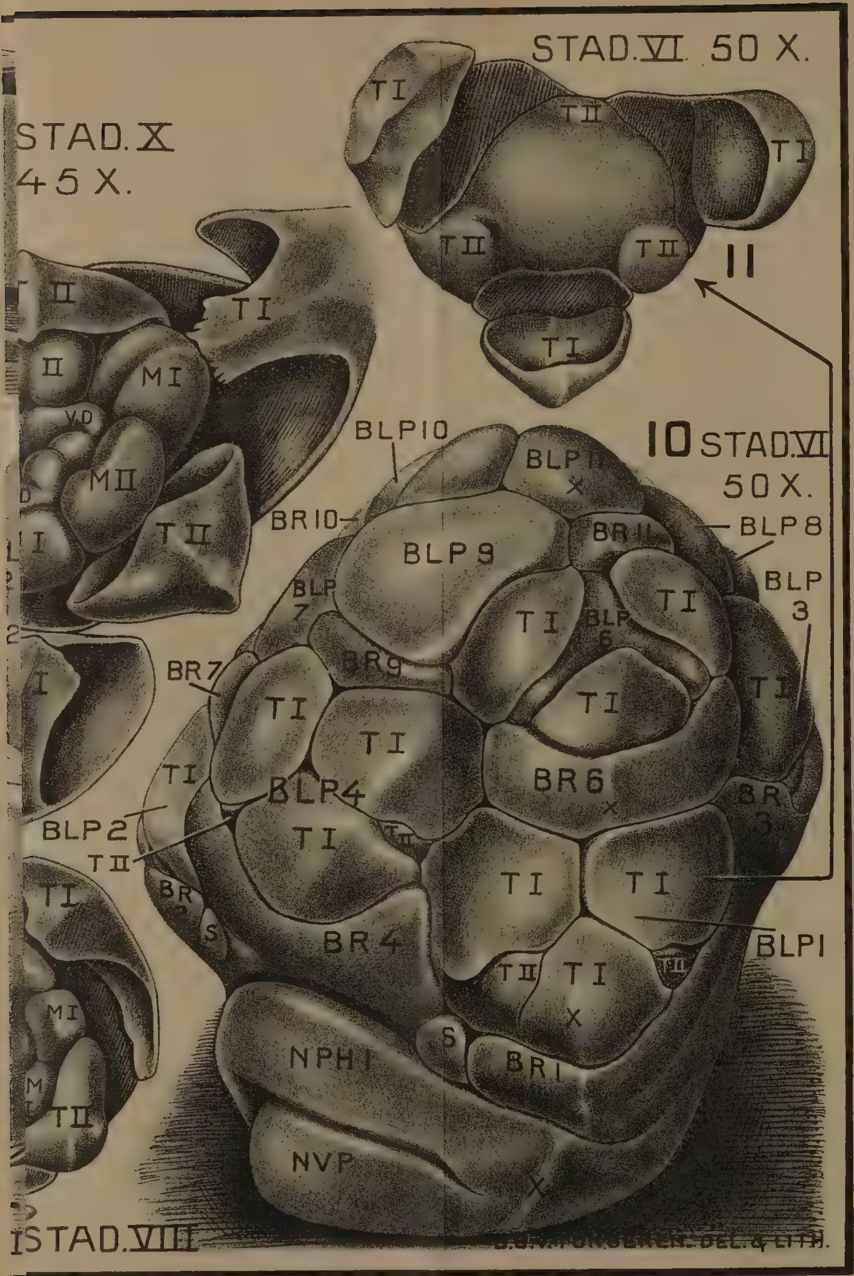

LIS. (L'INNOCENCE)

DRUK v. P.J. MULDER & Zn LEIDEN (HOLLAND)

12—13 cms (BLAAUW 1924, tab. 23) showed an optimum at 8 w. 25.5° + 4 w. 17°. It should be mentioned here, that in order to be compared, the bulbs must be of equal age, as is evident from comparison of tables 19 and 23 in BLAAUW 1924. For la Victoire (tab. 6*a*) a preliminary exposure to 20° for a long period (up to 12 weeks) is favourable, but for the combination of the optima, especially with a view to increase in weight, we shall have to raise the temperature (23°, still better : 25.5°), but a still higher temperature is markedly detrimental to the number of flowers.

Optima for the increase in weight (tabs. 7 and 7*a*) for the two varieties lie at 25.5° for 5 weeks and next 17° (besides for l'Innocence at 28° for 3 weeks and next 13° or 17°). An after-treatment in 17° is desirable for those. On comparing with Queen of the Blues (see BLAAUW 1924, tabs. 8 and 26), we are inclined to say that these early varieties should be exposed to the high temperatures for a shorter period than this Queen.

Final conclusion. For an optimal field-culture ca 25.5° for *eight weeks* and next 17° has appeared to be the best treatment for the late variety Queen of the Blues. The early varieties l'Innocence and la Victoire we had probably better expose to 25.5° *for a shorter period* (e.g. 5 weeks) and next to 17°.

This is also proved (with l'Innocence) by the fact that the flower-cluster after 8 weeks in 23° and 25.5° has already progressed so far, as is evident from the curve of dots (fig. 15).

We emphasize that we refer to healthy bulbs. For severely diseased sets quite a different treatment may be desirable as a cure.

Wageningen, October 1927. *Laboratory for*
Plant-physiological Research.

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————————, 1924. The results of the temperature during flower-formation for the whole Hyacinth. (2nd part). Proceedings of the Roy. Acad. of Sc. of Amsterdam, Vol. **27** and Commun. N⁰. 11 Laboratory f. plantphysiol. Research, Wageningen (Holland).

Miss I. LUYTEN, Miss G. JOUSTRA and A. H. BLAAUW, 1925. The results of the temp.-treatment in summer for the Darwin-tulip (Second part). Proc. Roy. Acad. of Sc. of Amsterdam, Vol. **29**, 1925 and Commun. N⁰. 18, Laboratory f. plantphysiol. Research, Wageningen (Holland).

Physics. — *The influence of pressure on the electrical conductivity of gold up to 1000 atmospheres.* By A. MICHELS and P. GEELS. (18th communication of the "VAN DER WAALS-Fund.") (Communicated by Prof. J. D. VAN DER WAALS.)

(Communicated at the meeting of October 29, 1927).

These measurements complete an investigation on the influence of pressure on the conductivity of gold, the preliminary results of which were given in the 17th communication [1].

It must be repeated that the material used in this investigation was gold-wire in a hard drawn state so that the influence of tempering might afterwards be investigated.

The method of measurement.

The bridge circuit depicted in figure 1, which has been described by F. E. SMITH [2], was used in place of the WHEATSTONE bridge used in the preliminary investigation.

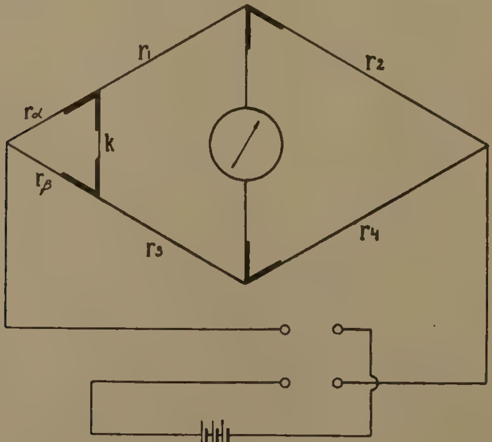

Fig. 1.

The peculiarity of this circuit, like that of the THOMPSON-bridge, is that the influence of the resistances of the leads is reduced to below the accuracy of observation.

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[1] These Proceedings 30, (1927) 47.

[2] Phil. Magazine 1912. Bd. 24. p 562.

As it is hoped to return to the advantages and disadvantages of this method, as well as to the best choice of the resistances, in a later communication, it will be sufficient to state here that :

$$\frac{r_\alpha}{r_\beta}=\frac{r_1}{r_3}=\frac{r_2}{r_4},$$

r_α and r_β being made of the order of 1000 ohms, whilst k was kept as small as possible ; r_1 was the measuring wire, and variations in the temperature were compensated by a gold wire r_3, which was not put under pressure (l.c.).

r_2 and r_4 are the coils of standard resistances of 1000 ohms with a calibrated variable resistance introduced into one of each pair of leads for regulation and for the compensation of the pressure effect.

Following the usual method, the coils of the standard resistances were commutated in each measurement in order to compensate for any irregularities in the resistances of the two coils. (This appeared necessary as deviations of the order of 5×10^{-6} were obtained, due to differences in the temperature coefficients.) The deflection of the galvanometer was observed by commutating the bridge current, the effects of thermal currents being thus eliminated.

The sensitivity.

The sensitivity of the circuit was sufficient to cause an alteration of resistance of 4×10^{-7}, when an ordinary MOLL galvanometer and a measuring current of 3 milliamps was used. This corresponds to a temperature difference of the measuring wire of 10^{-4} degrees.

Calculation showed that the increase in temperature of the gold wire resulting from the above measuring current, would not give an error greater than this.

The accuracy of the pressure measurements up to 250 atmospheres was greater than that of the resistance measurements.

As the new pressure balance for higher pressures had not been completed, the old, although improved, high pressure balance of the "VAN DER WAALS Fund" had to be used for 250—1000 atmospheres, and an accuracy cannot be claimed of more than 1 in 1000.

The observations were made in two series, one from 0—250 atms., with pressure intervals of 50 atms., and the other from 0—1000 atms. with pressure intervals of 200 atms. A few points overlap, and provide a good connection between the two series.

The measuring wire was once heated for a considerable time at 40° between two sets of measurements from 0—250 atmospheres.

In order to economise space all the results are not communicated.

The results of the measurements.

The tables below give:

Table I a series of measurements up to 250 atms. before the tempering at 40°;

Table II a series after the tempering;

Table III the results over the pressure range 0—1000 atms.

The tables are subdivided, and give under:

a. the total change of resistance per original Ohm;

b. the change of resistance per Ohm per kilogram, per sq. centimetre for the various pressure intervals.

TABLE I*a*.

$$-\frac{\Delta w}{w} \times 10^6.$$

Pressure in Kg/cm^2	51.5	101.5	151.5	201.5	251.5
16.95°	258^6	503^6	742^5	977^1	1206^1
28.46°	252^5	493^4	729^1	959^2	1185^5

TABLE I*b*.

$$-\frac{\Delta w}{\Delta p \, . \, w} \times 10^6.$$

Pressure in Kg/cm^2	0—51.5	51.5—101.5	101.5—151.5	151.5—201.5	201.5—251.5
16.95°	5.02	4.90	4.78	4.69	4.58
28.46°	4.90	4.82	4.71	4.60	4.53

TABLE II*a*.

$$-\frac{\Delta w}{w} \, . \, 10^6$$

Pressure in Kg/cm^2	51.5	101.5	151.5	201.5	251.5
18.75°	255^6	497^9	734^2	963^6	1183^6
18.87°	255^5	497^3	733^8	964^2	1183^6
28.5°	251^8	487^1	713^2	929^0	1136^4
28.2°	251^3	489^0	$71\;\;^8$	930^8	1136^9
40.0°	244^8	469^0	677^7	878^6	1071^0

TABLE II*b*.

$$-\frac{\Delta w}{\Delta p\, w} \,.\, 10^6$$

Pressure in Kg/cm²	0—51.5	51.5—101.5	101.5—151.5	151.5—201.5	201.5—251.5
18.75°	4.96	4.85	4.73	4.59	4.40
18.87°	4.96	4.84	4.73	4.61	4.39
28.5°	4.89	4.71	4.52	4.32	4.15
28.2°	4.88	4.75	4.52	4.32	4.12
40.0°	4.75	4.48	4.17	4.02	3.85

TABLE III*a*.

$$-\frac{\Delta w}{w} \,.\, 10^6$$

Pressure in Kg/cm²	209.4	408.3	607.1	806.0	1004.8
20.9°	994^8	1819^1	2506^9	3118^4	3699^5
28.3°	968^2	1763^1	2424^7	3029^6	3609^3
39.1°	917^8	1668^0	2298^0	2894^3	3473^3

TABLE III*b*.

$$-\frac{\Delta w}{\Delta p\, w} \,.\, 10^6$$

Pressure in Kg/cm²	0—209.4	209.4—408.3	408.3—607.1	607.1—806.0	806.0—1004.8
20.9°	4.75	4.14	3.46	3.07	2.92
28.3°	4.62	4.00	3.33	3.04	2.92
39.1°	4.38	3.77	3.17	3.00	2.91

Discussion of the results.

1^0. A deviation of 1.3×10^{-6} from a mean was obtained in one case in the measurements below 250 atmospheres, the remainder of the measurements showing a much better agreement.

As was expected, the measurements with the large pressure-balance did not give such good results, the maximum deviation from the mean amounting to 4×10^{-6}.

The deviations correspond to a temperature variation of $^1/_{3000}{}^\circ$ for the lower pressures, and $^1/_{1000}{}^\circ$ for the higher.

2^0. The tempering of hard drawn wire for a considerable time at 40° appears to have a very noticeable influence on the pressure effect.

In connection with this it must be mentioned that, although the wire had previously been compressed to 1300 atms. with the result that the hysterisis phenomena did not appear in the normal measurements, these phenomena reappeared after the tempering, but were diminished after first raising the pressure.

It thus appears almost impossible to compare the results of work on the pressure influence on metal resistances, unless the history of the wire is given accurately. All that is known of the temperature treatment of the gold wire used by BRIDGMAN is, that it was tempered between 0° and 140°—150°. BRIDGMAN, therefore, used a more strongly tempered metal, and this is in agreement with the pressure coefficient found by him (between 0—1000 atms. mean 3.1 $\times$ 10^{-6} at 250), for the latter deviates considerably from the present measurements in the same direction as does the coefficient found after tempering to 40°. In view of the tempering mentioned, it is doubtful whether BRIDGMAN determined his coefficients with sufficient reproducibility. Also, it is noteworthy that BRIDGMAN does not give a single intermediate determination between 0 and 1000 atmospheres, and that he has only attempted to determine the curve by a sort of extra-interpolation method, whilst the present measurements indicate, that the curve is most important in the region of lower pressures.

Again, BRIDGMAN did not observe any noticeable hysterisis phenomena such as is mentioned above.

A graphical representation of the present results is given in Fig. 2, and

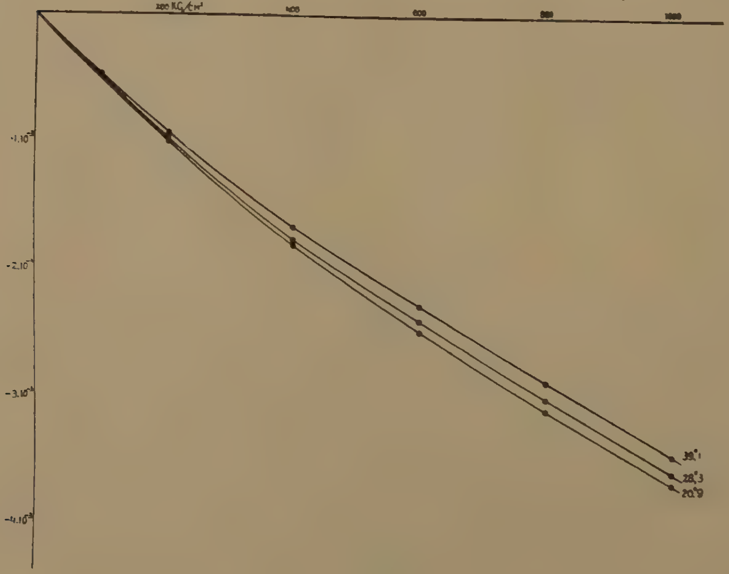

Fig. 2.

the deviation from the linear equation of the pressure effect, i.e. the deviations from the straight line drawn between the first and last points of Fig. 2, are plotted in Fig. 3.

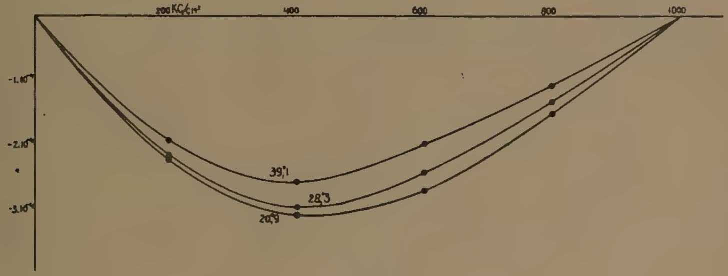

Fig. 3.

As BRIDGMAN has also published a graph similar to Fig. 3 it is possible to compare his and the present results.

Geology. — *On the age of Alkaline rocks from the island of Timor.* By Prof. H. A. BROUWER.

(Communicated at the meeting of November 26, 1927).

Among the Permian sediments of the island of Timor which consist principally of tuffs, marls with tuffaceous material, marls, limestones and effusive rocks were also found locally conglomerates with pebbles of eruptive material. The pebbles from these sediments were not yet examined microscopically and their description is not included in my paper on the rocks of East-Netherlands-Timor [1]). Permian conglomerates with pebbles of eruptive material have been described also from Djambi. TOBLER [2]) describes from there a thick series, consisting of porphyries and quartz-porphyries, keratophyres and quartzkeratophyres with tuffaceous sandstones and thick beds of conglomerate in which the pebbles consist of the same volcanic rocks.

The Timor conglomerate, of which the pebbles have been studied, has been found in the neighbourhood of the path Soefa-Maubesi, where this path passes the grass-grown hills which can be followed from the mountain Somohole in a northerly to north-easterly direction. The locality is near and to the N. E. of the most northern of these hills, where the path turns round it. Tuffaceous material and lapilli are found in many of the highly fossiliferous Permian sediments of this region while several eruptive rocks have been already described [1]). These latter rocks are often interstratifed with the Permian, they are partly effusive rocks, partly perhaps intrusive sheets. The top of the mountain Somohole consists for instance of medium-grained augite-amphibole-diorite and farther to the north quartz-bearing augite-amphibole-diorite have been found. The latter rock has a peculiar chemical composition, as I already have shown (loc. cit. p. 23); it differs from the normal quartz-diorites by its high Na_2O content and its low Al_2O_3 content, characteristics which point to a transition to alkaline rocks.

Gabbros, diabases and porphyrites have also been described from this region.

Typical alkaline rocks are found in adjacent regions. Of the alkali-rhyolites, alkalitrachytes and keratophyres, which have been described in my previous papers [3]), an alkalirhyolite is found in the Fatu Forfaik to the south-west of Sufa. Between the bivouac Sufa and the grass-grown hills to the north of the Somohole we found a brecciated alkalirhyolite with aegirine in the groundmass; alkalitrachytes and keratophyres have been found at different places in East-Netherlands-Timor. Many alkaline rocks are found in close connection with Permian sediments, we only mention the camptonites of the Noil Tonini, the alkalirhyolite of the Fatu Forfaik near Sufa, which rests upon Permian crinoidal limestone and the alkalitrachyte in the valley of the Noil Musa, which is found with Permian rocks.

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[1]) H. A. BROUWER. Gesteenten van Oost-Nederlandsch Timor. Jaarboek v. h. Mijnwezen in Ned. Oost-Indië. Verhand. 1916. I.

[2]) A. TOBLER. Djambi Verslag. Jaarboek v. h. Mijnwezen in Ned.-Indië. Verhand. 1919. III.

[3]) H. A. BROUWER. Neue Funde von Gesteinen der Alkalireihe auf Timor. Centralbl. f. Min. Geol. und Pal. 1913, p. 570; 1914, p. 741.

Constituents of the Permian conglomerate.

Several pebbles of which some are up to several centimeters in diameter were studied under the microscope. Partly they are typical effusive rocks, partly they are medium-grained or porphyritic with a holocrystalline groundmass. Many of the pebbles are strongly weathered and their identification is difficult, because the dark minerals and sometimes also the feldspar are entirely decomposed. It is probable that more alkaline rocks are found among them but they cannot be identified with certainty without a chemical analysis, because the typical minerals cannot be determined.

Quartz (alkali) syenite porphyry.

This rock consists of phenocrysts of microperthitic feldspar and of acid plagioclase (principally albite) in a groundmass consisting of the same feldspars, rather much quartz, much ore (and ore-bearing substance), also some calcite. The original dark minerals of the rock is transformed in secondary minerals. The character of the feldspar shows however sufficiently the high alkali-content of the rock, so that it can be brought with great probability to the quartz-bearing alkali-syeniteporphyries.

(Alkali) Syenite.

It is a fine- to medium-grained rock, consisting mainly of microperthitic feldspar with decomposed dark minerals, numerous small crystals of ore and apatite. The original composition of the dark minerals cannot be identified, they are completely altered to calcite and a substance rich in iron. Their forms sometimes have a resemblance to those of pyroxene, which perhaps contained the aegirine-molecule in connection with the large quantity of ferrugineous weathering-products.

(Alkali) Trachytes.

A great many of the pebbles, which were studied microscopically, are reddish to brownish rocks, which are rich in secondary minerals, principally calcite and also ferruginous weathering-products. The phenocrysts are sometimes abundant but they are always entirely altered, the feldspar generally into calcite, the original dark minerals into calcite with much of a ferruginous substance. In the groundmass, which is also rich in secondary products, numerous small feldspar-laths can often be recognized. Their extinction-angle is always small, orthoclase and acid plagioclase cannot be distinguished. Typical trachytic structures are characteristic for several of the rocks. It is possible, that andesitic rocks occur among those, which are strongly altered.

Quartzalkalitrachytes.

These are the only pebbles of the Permian conglomerates, in which the dark minerals can still be determined. They are confined to the groundmass, but certainly belong to aegirine-augite or aegirine, so that there can be no doubt, that these pebbles belong to alkaline rocks.

The phenocrysts are orthoclase or anorthoclase, they are partially altered, calcite is the principal secondary mineral. The groundmass is rich in ferruginous weathering-products and consists of feldspar, rather much quartz, ore and small elongated crystals, which, when they are not decomposed, show a distinct greenish colour. They are optically negative, have a small extinction-angle and belong to aegirine-augite or aegirine.

Although these rocks are altered rather strongly, the unaltered character of a part of the aegirine in the groundmass makes their determination possible with certainty, while the unaltered parts of the other rocks allow a probable determination only.

The pebbles of alkaline rock in the Permian conglomerates prove, that these rocks are not younger than Permian. In connection with the abundance of effusive material in the Permian sediments of the adjacent regions, while pebbles of eruptive rocks are the only ones, which have been found, and in connection with the fact that similar alkaline rocks have been found closely connected with Permian sediments, the conclusion that the alkaline rocks are of Permian age seems to be warranted.

Botany. — *The influence of growth-promoting substances on decapitated flower-stalks of Bellis perennis.* By INA E. UYLDERT. (Communicated by Prof. F. A. F. C. WENT.)

(Communicated at the meeting of November 26, 1927)

The fact is well-known that the growth of flower-stalks is greatly retarded by the removal of the flower-buds. Sometimes the growth may even come to a complete standstill.

SÖDING (1926) showed that this retardation does not take place when the flower-bud is fixed again on the stalk by means of gelatine.

CHOLODNY (1924) removed the tops of Lupinus hypocotyledons. In also removing the central portion of the organ, he obtained hollow cylinders incapable of geotropic curvature. If the coleoptile tips of Zea mays were placed in these stumps, the curvature could be induced again. In this way he stimulated the organs to resume their growth.

In view of these experiments it seemed feasible to induce growth in decapitated flower-stalks by means of growth-substances from other plants. The experiments described in this paper were carried out with the decapitated flower-stalks of Bellis perennis and growth-substances prepared from the coleoptiles of Avena sativa by Dr. F. W. WENT and Mr. H. E. DOLK.

Intact plants of Bellis were planted in zinc boxes. The boxes were placed in a glass-covered glass basin and placed in the green house. In this way they were kept in a moist atmosphere at a temperature of about 20° C. Large flowerbuds were cut off with a rasor, 5 to 7 millimeters below the bud. The remaining stalk was marked with India ink 15 millimeters below the wound. Three series of growth measurements were carried out.

A. By shoving a tightly fitting glass capillary over the stumps, the buds could be fitted on the stump with a drop of 15 % gelatine.

B. By sealing the capillary with wax on one side, and filling them with agar which contained the growth-substances one could observe the influence of foreign growth-substances.

C. Pure agar was used as control.

The increase in length was measured with a millimeter scale ; tenth of millimeters were estimated.

In series A the flower-buds were replaced.

In series B agar with growth-substances from Avena was put on the

TABLE I (flowerbuds, cut off with 7 millimeters of flower-stalk)

	A. flowerbuds increase in length in millimeters in 24 hours			B. agar with growth substances increase in length in millimeters in 24 hours			C. Control increase in length in millimeters in 24 hours	
1	2.6		1	0.7		1	0.2	
2	0.5		2	0.4		2	0.4	
3	1.1	average	3	1.3	average	3	0.2	average
4	0.7	1.36	4	1.5	1.09	4	0.5	0.39
5	0.8		5	1.2		5	0.6	
6	2.8		6	1.5		6	0.5	
7	1.0		7	1.0		7	0.3	

The increase in length of a 15 millimeter zone was measured.

flower-stalks. The quantity of growth-substances amounted to 3500 tip-minute [1]) per flower-stalk.

Serie C were the control experiment without flower-buds or growth substances.

This table shows that the flower-stalks in column B have grown considerably more than in column C and as much or a little less as in column B.

TABLE II (flowerbuds cut off with 5 millimeters of flower-stalk)

	A. flowerbuds increase in length in millimeters in 24 hours			B. agar with growth substances increase in length in millimeters in 24 hours			C. Pure agar increase in length in millimeters in 24 hours			D. Control increase in length in millimeters in 24 hours	
1	1.8		1	5.0		1	0.2		1	0.1	
2	2.2		2	1.8		2	0.7		2	1.1	
3	1.8	average	3	1.3	average	3	0.4	average	3	0.9	average
4	1.0	1.43	4	1.6	1.93	4	0.2	0.40	4	0.6	0.65
5	1.2		5	1.1		5	0.5		5	0.7	
6	0.6		6	0.8		6	0.4		6	0.5	

The increase in length of a 15 millimeter zone was measured.

---

[1]) One "tip-minute" is the amount of growth-promoting substances, which diffuses out of the coleoptile-tip into an agar disc in one minute. 600 tip-minute may mean therefore the amount diffusing out of 6 tips in 100 minutes or out of 12 tips in 50 minutes.

In series A flower-buds were replaced.

In series B agar was put on with growth-substances, 2000 tip-minute per flower-stalk.

In series C pure agar, without growth-substances, was put on.

D is another control in which the buds were simply cut off.

Then again pure agar was put on 12 flower-stalks (A) of 6 flower-
The increase in length of a zone of 15 millimeters in 24 hours amounted
stalks (B) the flower-bud was simply cut off.
to 0.38 millimeters at A and 0.33 millimeters at B as an average.

This shows that in harmony with table II, pure agar has no influence on the growth.

Therefore the result of this experiment is, that flower-buds as well as growth-substances from Avena accelerate the growth considerably.

The table shows the remarkable fact, that the increase in length of B 1 was exceedingly great. It may be observed that the increase in length with normal plants measured over the same zone, in the same time, amounts to 3—10 millimeters. It therefore appeared possible to make use of such experiments to demonstrate the presence of growth-substances. For, at the beginning of the experiment, it was unknown to me, in which agar the growth-substances were present; I received the information after the measurements were completed.

The growth-substances present in the coleoptiles of Avena and the flower-buds exert an analogous influence on the flower-stalks of Bellis perennis; they both accelerate the growth.

Botanical Laboratory.

Utrecht, November 1927.

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Chemistry. — *The Essential Oil from Gastrochilus panduratum Ridl.* By A. J. ULTÉE. (Communicated by P. VAN ROMBURGH.)

(Communicated at the meeting of December 17, 1927).

The rhizome of this plant, which belongs to the Zingiberaceae family, is characterised by the presence of an essential oil possessing a fresh, pleasant aroma, resembling to some extent the smell of Esdragon and Basicilum oil.

As this rhizome constitutes an ingredient of an Indian rice dish, it is obtainable at the so-called "pasars" (under the Malayan name *Temoe koentji*) but only in small amounts, so that considerable patience was required to collect sufficient material for the examination of this hitherto uninvestigated oil.

The yield of the essential oil is also very small, and varied from only 0.06 % (by weight) in the young fresh rhizomes, to 0.32 % in older material, which however contained much less water.

The specific gravity of the freshly distilled oil varied according to the different methods of extraction, from 0.8636 at 25° (young material) to 0.8731 at 31°. If the oil is allowed to remain exposed to the air, its consistency and specific gravity rapidly increase, a result that would almost certainly be caused by the oxidation of an olefinic terpene present in the oil. After some months preservation in a bottle which was repeatedly opened, the specific gravity had already risen to fully 0.9.

The oil was of a faint yellow colour and showed a dextro-rotation ; in a tube 10 cm. long, different preparations showed rotations from $+10°30'$ at 27° (young material) to $+12°56'$ at 32°.

The oil had a neutral reaction ; the presence of esters was determined by boiling the oil with alcoholic potash and subsequent back titration with sulphuric acid.

2.2460	grams oil	required	1.20	cm^3	0.5 n alcoholic potash.
1.7950	,, ,,	,,	0.99	,,	,, ,, ,,

If one assumes that the methyl ester of cinnamic acid (see below) is the only ester present, it is thus contained in the oil to an extent of 4.32 and 4.46 % respectively.

Since this ester has been isolated by previous investigators, amongst others VAN ROMBURGH, from several members of the Zingiberaceae, the oil was examined first for this ester. Methyl cinnamate is characterised by its extremely slow rate of distillation when distilled with steam.

100 cm^3 of the oil were thus steam distilled until the residue (10 cm^3) had about the same specific gravity as water. Upon fractionation of this

residue, the temperature rose at once to 220°, the greater part (3 cm^3) distilling between 250—270°. A thick brown residue remained in the flask and could not be further distilled without undergoing decomposition.

After repeated distillation, a small amount of distillate was obtained which boiled between 255—260° and which solidified upon cooling. It melted at ± 33°.

These properties agree with those of methyl cinnamate, so that it is practically certain that this compound is present in the oil.

The oil that had passed over during the steam distillation, was distilled at ordinary pressure, it distilled mainly between 160—220°, the greater part being collected between 180—200°. The specific gravity of the portion that first collected amounted to 0.8553 at 28°.

A resinous residue remained in the flask, and probably represented the oxidation product of an olefinic terpene, consequently during the further examination of the oil, the distillations were carried out under reduced pressure.

The fraction that boiled between 180—200° possessed a distinct smell of cineol, and since this compound is known to occur in various members of the Zingiberaceae, the oil was examined for it. The oil was shaken out with a 50 % solution of resorcinol, separated from the unabsorbed portion and steam distilled. In this way I obtained a liquid that floated upon the aqueous distillate, which after drying was distilled from sodium.

The liquid thus obtained had the following properties :

1. Boiling point : 176—177° ;
2. Specific gravity at 25° : 0.9226 ;
3. The addition product formed with iodol melted at ± 112°.

These properties show that cineol is undoubtedly present.

In order to obtain an idea as to the amount actually present, 5 cm^3 of the original oil were shaken out with the resorcinol solution, whereby 1.6 cm^3 were absorbed, corresponding to a cineol content of 32 %.

Since in this determination, oxygenated compounds, and thus methyl cinnamate, can influence the result, the figure obtained is probably on the high side.

In the fraction distilling over below 200°, and thus probably on the low side, 26.4 % of cineol was found.

During the vacuum distillation (35 mm) of the original oil, at 120° some crystals collected in the condenser. The fraction distilling between 120—140° was now distilled once more at ordinary pressure, up to 215°, and after strongly cooling the distillate, crystallisation took place. After suction filtration, pressing between filter paper, and sublimation, the crystals, which possessed a strong camphoraceous smell, melted at 175°.

When mixed with pure camphor, the melting point rose to 177°.

When thrown upon water, the crystals showed the typical phenomenon of camphor.

The occurrence of camphor in this essential oil is thus proven. The content is however very small and probably amounts to not more than a few percent.

The lower boiling fractions from the vacuum distillation, which had been freed from cineol by shaking out with resorcinol solution, and which contained the supposed olefinic terpene, were repeatedly distilled from sodium in an atmosphere of carbon dioxide in order to prevent resinification, this however, could not be completely avoided.

Finally, a very mobile oil having the following constants was obtained :

1. Boiling point at 13 mm 62—64° ;
2. Specific gravity at 17° 0.8253 ;
3. Refractive index $n_D^{12} = 1.4843$;
4. Optical rotation $\alpha_D^{12} = 11^\circ.42'$.

The boiling point at ordinary pressure (in an atmosphere of carbon dioxide) was 174° at 753 mm, but it altered quickly.

A boiling test carried out in conformity with the method employed by ENKLAAR (see thesis) for ocimene, gave the following results :

At the commencement	the boiling point was	174°
After 2 minutes	,, ,, ,, ,,	175
,, 3 ,,	,, ,, ,, ,,	176
,, 5 ,,	,, ,, ,, ,,	177.5
,, 10 ,,	,, ,, ,, ,,	178.5
,, 25 ,,	,, ,, ,, ,,	180
,, 60 ,,	,, ,, ,, ,,	183

The course followed is thus different from that of ocimene.

By distillation in vacuum (at 13 mm to 80°) a liquid having a specific gravity of 0.8349 at 24° was obtained.

The olefinic terpene was reduced with sodium and alcohol according to the method of ENKLAAR. After steam distillation it was distilled at ordinary pressure from sodium. A fraction was obtained that boiled between 168—169° at 754 mm. *It possessed a smell similar to that of hydromyrcene.*

Optical rotation : $\alpha_D^{12} = 12^\circ 54'$.

Specific gravity at 14° 0.805. Refractive index $n_D^{12} = 1.4553$.

The olefinic terpene (in cooled acetic acid) was treated with bromine in acetic acid solution. Hydrogen bromide was evolved and the liquid became coloured, but no solid compound was obtained.

In conclusion it may be mentioned, that the lower fractions obtained from the distillation of the olefinic terpene had a turpentine like smell and a higher specific gravity, so it is probable that another terpene is also present in this essential oil.

Anatomy. — *The influence of the cephalization coefficient and body size upon the form of the forebrain in mammals.* By C. U. ARIËNS KAPPERS.

(Communicated at the meeting of October 24, 1927).

In the following pages I give some figures concerning the general form of the forebrain in less and more cephalized mammals of the same orders, and some data concerning the influence of the body size upon this form in different varieties of the same species, or in different, but equally cephalized species of the same family.

As only the forebrain is concerned here, my indices should not be confused with those obtained by measuring the length-width index of the endocranial cavity, since in this cavity the cerebellum also is included.

Whereas in the anthropoids (also in several other monkeys) and in man, the length-width index of the endocranial cavity corresponds with that of the forebrain, which in these animals and in man covers the cerebellum entirely, it is different with lower mammals, where the forebrain covers the cerebellum only partly or not at all (Fig. 1) and its length therefore is not the same as that of the endocranial cavity.

As a standard for the greatest length of the forebrain I did not take the greatest horizontal measurement (*b*—*b′*) but the greatest possible distance between the front- and hindpole (*a*—*a′* in Fig. 1), which in most animals is an inclining line. The accompanying figure explains my intention. I did this because it gives a better expression of the size of the brain and its increase in length.

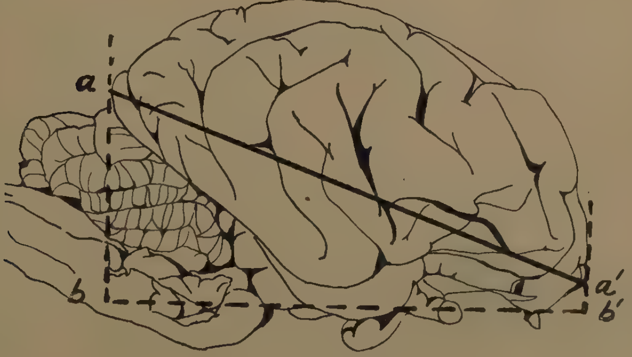

Fig. 1. Felis leo (nat. size).

In the following tables are given the greatest length and width of the forebrain and the index resulting therefrom in more and less cephalized species of the same orders and in some differently cephalized species of the same families. In these animals consequently, the cephalization coefficient k of Dubois' 1) formula, $E = kPr$, is obviously different.

I noted also the total weight of the brain up to 2—3 mm beyond the calamus. *The material used was weighed without meninges* 2) after preservation in formaline 10 % 3). In foot notes the brainweight found by others 4) is mentioned.

MARSUPIALS.

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Metachirus opossum (S.Am.)	2.7 cm	1.9 cm	70.4	3.75 gr.
Macropus cervinus	5.— „	4.6 „	92.—	61.— „
Macropus robustus	5.4 „	5.— „	92.6	64.— „
Phascolomys latifrons	4.2 „	4.4 „	105.—	37.5 „

Among marsupials (table 1), according to Dubois, a difference in the cephalization coefficient occurs between the North American (Virginian) opposum 5) and the Macropodidae, that of the latter being about 1½ × as large as that of the opossum. This apparently corresponds with a greater forebrain index, a distinct brachencephaly in the Macropodidae.

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1) Dubois. The proportion of the weight of the brain to the size of the body in mammals. Verhandelingen der Kon. Akad. v. Wetenschappen, Vol. **5**, N⁰. 10, 1897 (the same in Bulletin de la Société d'Anthropologie de Paris 1897 and Arch. f. Anthropologie Bnd. **25**, 1898).

Dubois. On the significance of the large cranial capacity of Homo neandertalensis. These Proceedings Vol. **23**, 1921,

Dubois. Phylogenetic and ontogenetic increase of the volume of the brain in Vertebrata. These Proc. Vol. **25**, 1922,

Dubois. The brainquantity of specialized genera of mammals. These Proc. Vol. **27**, 1924.

2) This may explain the slightly smaller weights of some of my specimens in comparison with those found by others.

3) According to Flatau the brainweight increases about 1 % after preservation (during 15 months) in 10 % formaline. (Beitrag zur technischen Bearbeitung des Zentral-Nervensystems, Anat. Anzeiger, Bnd. **13**, 1897).

4) Specially the following: Kohlbrugge, Zool. Ergebn. einer Reise in Niederl. Ost Indien, Leyden 1891, p. 139 and Natuurk. Tijdschr. v. Ned. Indië, Deel **55**, p. 261, Batavia 1896, Zoogdieren v. d. Tengger. Further: Zeitschr. f. Morph. und Anthrop., p. 43—55, Bnd. **2**, 1890, and Monathschr. f. Psychiatrie und Neurologie, 1901.

Weber, Vorstudien über das Hirngewicht der Säugethiere, Festschr. f. Carl Gegenbaur, Leipzig 1896 and these Proceedings 1896.

Ziehen, Anatomie des Zentralnervensystems, Fischer, Jena, 1899.

Spitzka, Brainweight of Animals etc. Journ. of Comp. Neurol., Vol. **13**, 1903.

5) Didelphis virginiana has a brainweight (Ziehen) of 3.9—4.5 gr.

The remarkable brachencephaly of the Wombat is not a result of a greater cephalization compared with the Macropodidae, but is due to flatness of the front part of its skull, which gave it the epithet "latifrons".

In Rodents with various cephalization coefficients the following proportions were found:

RODENTIA.

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Cavia cobaya	2.24 cm	2.10 cm	93.7	4.3 gr.
Dasyprocta aguti	3.44 „	3.35 „	97.4	17.5 „ [1]
Hydrochoerus capybara	5.69 „	5.69 „	100.—	58.— „ [2]
Dolichotis patagonica	3.8 „	4.55 „	120.—	31.5 „

Here also, the increase of brachencephaly in animals with a greater cephalization coefficient is obvious, since in Hydrochoerus and Dolichotis this coefficient is more than twice as large as in Cavia aperea, the wild form of Cavia cobaya.

The same is observed if we compare the poorly cephalized artiodactyle, Sus scrofa, with the more cephalized artiodactyles, the giraffa and dromedary, and with the perissodactyle horse. The cephalization coefficients of the giraffa, dromedary and horse are twice as large as that of Sus. It is obvious that the last three have also a higher index. Upon the very high index of the dromedary, the youth of this specimen also has influence (compare page 76).

UNGULATES.

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Sus scrofa dom.	7.28 cm	5.68 cm	78	112.— gr. [3]
Bos taurus	10.6 „	10.09 „	95.2	403.— „ [4]
Camelopard. giraffa	13.5 „	11.96 „	88.6	665.— „ [5]
Camelus dromed. (young sp.)	9.8 „	10.— „	102.—	478.— „ [6]
Equus caballus	10.79 „	10.17 „	94.2	549.— „ [7]

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[1] SPITZKA found variations of 16—21 gr. WEBER found 20 gr.

[2] A specimen weighed by WEBER (apparently larger) showed 75 gr.

[3] ROGNER (quoted from ZIEHEN) found variations from 111—120 gr.

[4] BISCHOF found variations from 400—500 gr.

[5] WEBER's largest specimen had a brainweight of 680 gr.

[6] LAPICQUE weighed a dromedary cerebrum of 650 gr., ZIEHEN one of 655 gr.

[7] WEBER found 615 gr., SPITZKA only 519.5 gr. COLIN 593—640 gr.

The brachencephaly of the *elephant* is striking, especially if we compare this animal with its nearest relative among living ungulates, *Procavia dorsalis*, but also in comparison to the Tapir. This is again in accordance with the greater cephalization of the elephant which, according to DUBOIS, is about 4 × greater than its nearest living relative, the ungulate Procavia, and also more than the tapir.

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Procavia dorsalis	3.92 cm	3.1 cm	79.—	19.7 gr.[1]
Tapirus indicus	8.— „	7.7 „	96.2	204.9 „[2]
Elephas indicus	23.— „	28.5 „	124.—	5474.— „[3]

If we now study the Carnivora, Viverridae, Mustelidae, Procyonidae and Ursidae and among the Cetacea an Odontocete and a Mysticete, it is obvious that here again the more cephalized forms are more brachencephalic. So the Viverridae generally have the smallest cephalization coefficient.

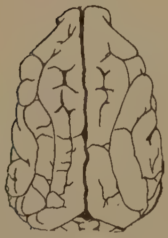

Fig. 2. Paradoxurus musanga, adult (nat. size)

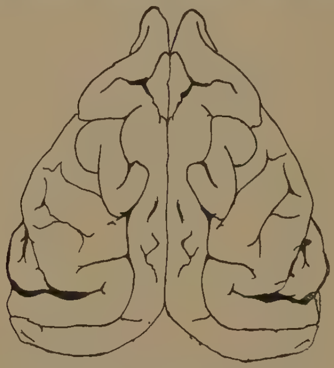

Fig. 3. Lutra vulg. adult, (nat. size)

CARNIVORES.

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Mungos mungo	3.4 cm	2.5 cm	73.5	12.— gr.[4]
Paradoxurus musanga	4.2 „	3.2 „	76.2	23.5 „[5]
Arctitis binturong	5.— „	3.9 „	78.—	37.— „

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[1] WEBER found with related Hyrax capensis variations of 19.2 gr. to 21 gr.

[2] WEBER found 265 gr.

[3] The heaviest specimen of CRISP had a brainweight of 5430 gr.

[4] Herpestes mungo by WEBER: 10.9 gr.

[5] WEBER: 22 gr.

CARNIVORES (*Continued*).

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Putorius putorius (1)	3.1 cm	2.3 cm	74.2	7.7 gr. (2)
Meles taxus	5.46 „	4.5 „	82.4	45.— „
Lutra vulgaris	5.8 „	5.— „	86.2	45.6 „ (3)
Nasua rufa	5.2 cm	3.71 cm	72.0	29.9 gr. (4)
Procyon cancrivorus	7.— „	5.7 „	81.4	93.— „
Ursus arctos (young spec.)	9.—cm	8.— cm	88.8	234.— gr. (5)
Ursus malayanus	9.4 „	8.5 „	90.4	313.— „ (6)
Ursus maritimus (young sp.)	11.— „	10.1 „	92.—	424.— „ (7)

CETACEA.

Balaenoptera sulfurea	22.—cm	29.8 cm	135	5676.— gr. (8)
Phocaena communis	7.8 „	12.4 „	160	389.— „ (9)

Only Paradoxurus and Arctitis have (DUBOIS) a 1¼ × larger cephalization coefficient than the Mustelid Putorius, and among Mustelids Meles and Lutra, have the greatest cephalization coefficient. My table shows that these two are also the most brachencephalic while Putorius is less brachencephalic than the Viverridae.

The largest cephalization coefficient of all carnivora, however, occurs in the Ursidae whose cephalization coefficient on the average is twice as large as that of the Viverridae (DUBOIS l.c. tertio p. 322) and 1½ × as large as that of the Nasuae. In accordance with this it appears that the Ursidae are also the most brachencephalic.

It is remarkable that Procyon cancrivorus (the crab washbear), which in the zoological system is more related to the Nasuae, also belonging to the Procyonidae, has a larger cephalization coefficient and accordingly a larger forebrain index than the Nasuae.

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(1) With the small Grison and Mephitis I found broader brains (80 and 82), notwithstanding their small cephalization coefficient. I ascribe this to the smaller size of these animals (Cf. p. 8 and 9).

(2) DUBOIS: 7.8 gr.

(3) SPITZKA found with Lutra 39 gr.; HUSCHKE (citated after ZIEHEN) 42—51 gr.

(4) SPITZKA found variations from 29 to 41 gr.

(5) SPITZKA found with Ursus Americanus variations from 192 to 248 gr.; WEBER found a cerebrum of Ursus arctos weighing 407 gr.

(6) WEBER found 325 gr.

(7) Ursus maritimus may, according to WEBER, reach a fresh brainweight of 530 gr.

(8) WEBER's specimen of Balaenoptera sibbaldi had a brainweight of 7000 gr.

(9) ZIEHEN found an average of 512 gr.

Among the Cetacea, the Odontocetes have a greater cephalization coefficient than the much bigger Mysticete Balaenoptera (DUBOIS). And again this corresponds with a greater brachencephaly in Odontocetes.

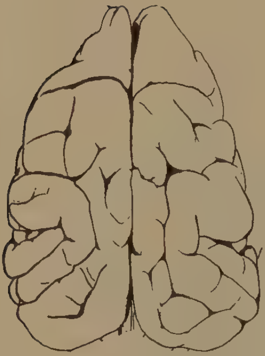

Fig. 4. Nasua rufa, adult (nat. size)

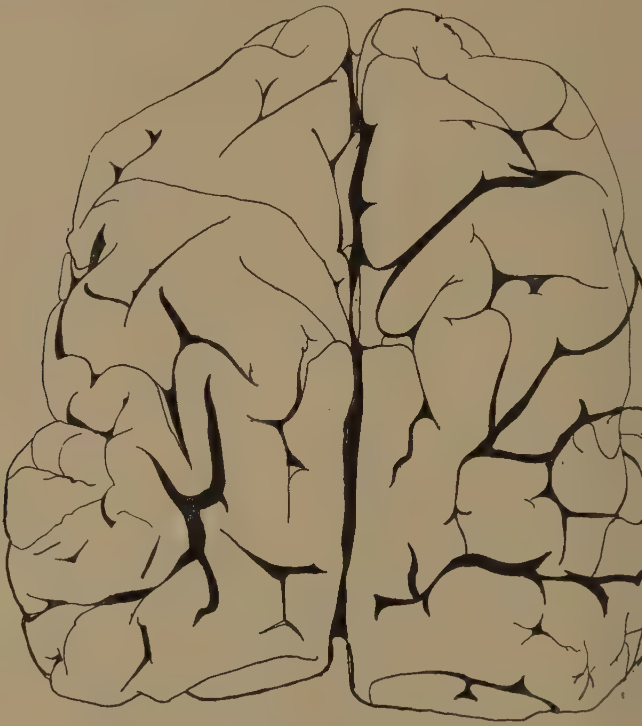

Fig. 5. Ursus maritimus, adult (nat. size)

PROSIMIAE.

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Lemur mongoz	4.53 cm	3.8 cm	83.8	23.5 gr.[1]
Chiromys madagascariensis	4.7 „	4.5 „	95.8	34.6 gr.[2]

Of the Prosimiae, Chiromys has a cephalization coefficient twice as large as the Lemurs. Correspondingly its forebrain is more brachencephalic.

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[1] WEBER found variations of 21—28 gr.

[2] WEBER found a brainweight of 43 gr.

WESTERN APES

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Callithrix jacchus	3.5 cm	2.46 cm	70.3	10.— gr. [1]
Cebus apella	7.— „	5.5 „	78.5	69.— „ [2]

In the Platyrrhine apes, the coefficient of Cebus is four times as large as that of Callithrix. Accordingly we see that its forebrain index is less dolichencephalic than that of Callithrix, although the latter has a smaller body and consequently one might expect a larger index in Callithrix (see p. 72, 73, 74).

In Katarrhine monkeys and in Anthropoids I found the following indices.

EASTERN APES.

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Cercopithecus pygerythrus	6.8 cm	5.5 cm	80.9	76.— gr.
Macacus nemestrinus	7.5 „	6.— „	80.—	103.8 „ [3]
Symphalangus synd. (small spec..)	7.6 cm	6.—cm	80.—	107.— gr. [4]

ANTHROPOIDS.

Pan (Troglodytes) Schweinf.	10.8 „	9.1 „	84.2	374.— „ [5]
Simia sat. (small spec.)	9.8 „	8.6 „	87.7	295.— „ [6]

The cephalization coefficient of these Anthropoids appears to be twice as large as that of the Katarrhine monkeys (DUBOIS) and accordingly they are more brachencephalic [7]. The Gibbon (Symphalangus syndactylus), which usually is considered as a transition between Cercopitheci and Anthropoids, stands closer to the first in this respect.

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[1] SPITZKA found variations of 7—9 gr. in Jacchus vulg.

[2] For Cebus capucinus WEBER also found 69 gr.

[3] SPITZKA found in Mac. nemestrinus an average brainweight of 110 gr.

[4] KOHLBRUGGE found variations from 102 to 130 gr.

[5] The Troglodytes of WEBER varied from 340—348 gr. One specimen of SPITZKA weighed 380 gr.; that of MÖLLER 391 gr.; one of MARSHALL 412 gr.

[6] The statements of WEBER vary from 306—339 gr. DENIKER and BOULART reported a cerebrum of 400 gr., just as MILNE EDWARDS. R. FICK, one of even 440 gr. (quoted from ZIEHEN).

[7] The index of the Gorilla brain and the endocranium varies very strongly. Two young cerebra of Madelle COUPIN gave me an average of 84.4, two endocranial casts received from ELLIOT SMITH 79,2. BOLK (these Proceedings 1925) found variations between 80.6 and 85.9 (an average of 83.25) and besides on specimen with an index of 72.2. HARRIS, having investigated the greatest number of endocrania, found variations from 72.1 to 86.8. (American Journal of physical anthropology, 1926).

From these comparisons it appears that generally the more highly cephalized species of an order tend to brachencephaly. Their forebrain has a rounder instead of a longer form. Deviations of this rule are discussed p. 78—80. This tendency must be explained by the fact that if a brain increases more than in the usual proportions with the body, its volume increases more than its surface, the latter being limited by the skull. Consequently it approaches the form of the globe, since a globe contains the greatest volume with the smallest surface.

As the more highly cephalized species in my tables are also generally larger animals, one might be inclined to consider body size as a factor, causing brachencephaly. That this is not so, however, clearly appears from the following tables, in which smaller and larger, but equally cephalized representatives of the *same species*, or of the *same family*, are compared, so that the influence of body size can be traced separately.

From these tables it appears that the larger Canidae, Felidae, Nasuae, Pinnipedia and Odontoceti are *less brachencephalic* than the smaller ones, so that the brachencephaly cannot be the result of a larger body. The same appears with the Giraffidae, Antilopes, Cervidae (excepted Cephalobus and Capreolus) and Equidae. And again this is observed in monkeys, if we compare equally cephalized species of different size.

In the following groups the cephalization coefficient *k* is about the same. The species differ only in size. It is obvious that in nearly all cases an *increase in body size* is accompanied with a *decrease of*

Influence of body size.

CANIS. [1]

Animal	Forebr. length	Forebr. width	Forebr. index.	Total brainweight
Griffon	5.6 cm	5.05 cm	90.2	55.2 gr.
Spaniel	7.1 „	5.75 „	81.—	90.— „
St. Bernard	7.4 „	5.70 „	77.—	97.— „ [2]
German dog	7.7 „	5.70 „	74.—	97.5 „

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[1] The weights which I found for these dogs are nearly all lower than those found by CORNEVIN (Études sur le poids de l'encéphale dans les diverses races des espèces domestiques. Journ. de Médicine vétérinaire et de Zootechnie, Année 1899 p. 248) and RÜDINGER, Ueber die Hirne verschiedener Hunderassen (Verhandl. der Anat. Gesellsch. in Strassburg 1894 p. 173). I presume that they made the weighings with the meninges (see note 2, p. 66).

[2] A very large specimen examined by WEBER had even a brain weight of 123 gr.

FELIDAE.

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Felis bengal. minuta	3.7 cm	3.5 cm	94.6	23.5 gr. [1]
Felis domestica	4.2 „	3.9 „	93.—	34.— „ [2]
Felis nebulosa	5.2 „	4.8 „	92.3	46.— „
Felis pardalis	5.7 „	5.18 „	91.—	72.— „
Felis concolor	7.4 „	6.25 .	84.5	149.5 „ [3]
Felis leo (small spec.)	8.3 „	7.3 „	88.—	195.— „ [4]
Felis tigris	8.8 „	7.4 „	84.1	202.— „ [5]

NASUA.

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Nasua narica	4.6 cm	3.6 cm	78.3	23.3 gr.
Nasua rufa	5.2 „	3.71	72.—	29.9 „

PINNIPEDIA.

Phoca vitulina (small spec.)	7.1 cm	8.3 cm	116.9	205.— gr [6]
Otaria jubata	9.7 „	10.1 „	104.—	268.— „ [7]
Zalophus californianianus	10.1 „	10.1 „	100.—	347.— „ [8]

ODONTOCETI.

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Phocaena communis	7.8 cm	12.4 cm	160	389.— gr. [9]
Tursio tursiops	12.85 „	16.85 „	132	± 1350.— ,, [10]

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[1] WEBER: 23.6 gr.

[2] WEBER found an average of 31 gr.

[3] A specimen of WEBER had a brainweight of 137.5 gr.

[4] WEBER found as average brainweight 216 gr.

[5] WEBER: 246 gr.

[6] WEBER found variations of 242—290 gr.

[7] These measures have been made on the drawing of MURIE (who gives also this weight) in his descriptive anatomy of the sealion (Otaria jubata). Transactions of the zool. Society of London. Vol. 8, 1874.

[8] SPITZKA found with this animal 335 gr. brainweight, WEBER 347—399 gr.

[9] ZIEHEN found an average brainweight of 512 gr.

[10] WEBER weighed a cerebrum of 1886 gr. It is remarkable that my specimen of Balaenoptera sulfurea (a Mysticete) has a somewhat larger index than Tursio. Possibly the heavy cerebrum of my Balaenoptera is a little broadened by lying on its basis.

ANTILOPES.

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Gazella dorcas	6.1 cm	5.5 cm	90.1	78.— gr.
Gazella soemmer.	7.5 „	6.7 „	89.3	128.5 „
Cephalophus maxwelli	*5.35* „	*4.7* „	*87.8*	*47.—* „ [1]
Damaliscus albifrons	9.12 „	7.87 „	86.3	254.5 „
Taurotragus oryx liv.	12.2 „	10.3 „	84.4	437.— „
GIRAFFIDAE.				
Ocapia Johnstoni	10.6 cm	10.— cm	94.—	450 gr. [2]
Camelopard. giraffa	13 5 „	11.96 „	86.6	665 „

brachencephally or an increase of *dolichencephali.* This corresponds with the phenomenon found by KLATT [3]) that larger animals of the same family have longer skulls.

Consequently the greater brachencephaly in more cephalized species is a result of their greater cephalization only.

The elongation of the brain in larger animals of the same species is a phenomenon analogous to what is seen in onto-

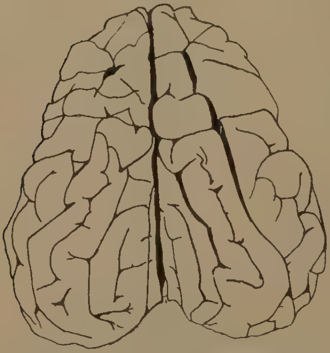

Fig. 6. Cervulus muntjac adult (nat. size).

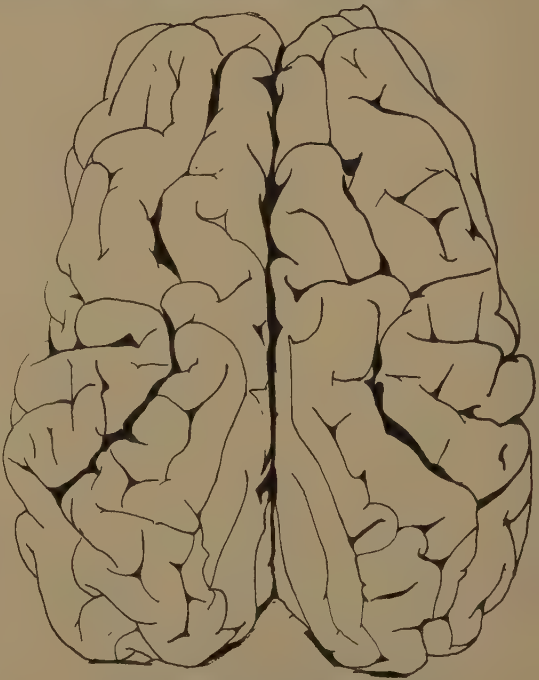

Fig. 7. Rucervus duvauc. adult (nat. size).

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[1] WEBER's heaviest cerebrum weighed 41.1 gr.

[2] After the statement of BLACK: Journ. of Comp. Neurol. Vol. **25**, 1915.

[3] KLATT. Ueber den Einflusz der Gesamtgrösze auf das Schädelbild nebst Bemerkungen über die Vorgeschichte der Haustiere. Archiv für Entwicklungsmechanik. Bnd **36**, 1913.

CERVIDAE.

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Cervulus muntjac (small sp.)	5.23 cm	4.49 cm	85.8	44.— gr.
Rusa porcinus	8.2 „	7.— „	85.4	143.— „ 1)
Dama dama	8.22 „	6.99 „	85.0	167.— „
Rusa hipp. molucc.	8.34 „	7.1 „	85.1	186.— „
Capreolus caprae	*6.79 „*	*5.70 „*	*83.9*	*85.— „* 2)
Rucervus eldi	9.65 „	7.4 „	79.3	202.— „
Rucervus duvauceli	10.7 „	8.3 „	77.5	297.— „
EQUIDAE.				
Equus asinus	9.2 cm	8.74 cm	95.—	334.7 gr. 3)
Equus caballus	10.79 „	10.17 „	94.2	549.— „ 4)
SIMIAE.				
Cercopithecus cynosurus	5.9 cm	4.8 cm	81.3	52.— gr. 5)
Cercopithecus pygerythrus	6.8 „	5.5 „	80.9	76.— „
Macacus rhesus	6.8 cm	5.57 cm	82.—	76.5 gr. 6)
Macacus nemestrinus	7.5 „	6.— „	80.—	103.8 „ 7)

genetic development, as appears from the ontogenetic tables (comp. also fig. 8 with 3 and 9 and 10).

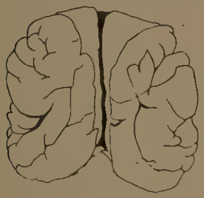

Fig. 8. Lutra vulg. neon. nat. size). Comp. this figure with fig. 3.

The same observation is made by MANOUVRIER and by M$^{\text{elle}}$ COUPIN for the Gorilla and by the latter also for the Chimpanzee and Cercopithecus 8). I have mentioned elsewhere that the same occurs in man 9).

The fact that the length of the brain generally increases more than the width, in the individual development as well as in the larger representatives

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1) WEBER: 142 gr.

2) WEBER found in Cervus capreolus 98 gr. HUSCHKE (quoted from ZIEHEN) 94 gr.

3) COLIN found with Equus asinus (quoted from ZIEHEN) 385 gr.

4) WEBER: 615 gr., COLIN: 593—640 gr.; SPITZKA 519.5 gr.

5) SPITZKA: 68.5 gr., WEBER: 70.5 gr.

6) SPITZKA found with Macacus rhesus an average brainweight of 80 gr. with variations of 71 to 98 gr.

7) SPITZKA found with Macacus nemestrinus an average brainweight of 110 gr. with variations of 84 to 128 gr.

8) F. COUPIN. Le développement comparé du cerveau chez l'homme et chez les singes. Revue scientifique 1926 p. 9 and fig. 2. It is remarkable that Mad$^{\text{elle}}$ COUPIN could not state this with Semnopithecus, similarly I did not find it in Bos taurus.

9) ARIËNS KAPPERS. Indices for the anthropology of the brain applied to Chinese, dolicho- and brachycephalic Dutch, foetuses and neonati. Proceedings Kon. Akademie. Amsterdam, Vol. **30**, 1926.

of the same species or family cannot be explained only by abscence of necessity of the brain in larger animals to tend to globular form. This appears from the fact that among Cetacea, the brain which in smaller forms is flattened antero-posteriorly, in larger forms approaches the globular form because the index decreases in the direction of hundred.

CARNIVORES.

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Lutra vulg. neon.	2.6 cm	2.7 cm	104.—	6.6 gr.
„ „ adult.	5.8 „	5.— „	86.2	45.6 „
Felis dom. neon.	2.—cm	2.1 cm	105.—	4.15 gr.
„ „ adult.	4.2 „	3.9 „	93.—	34.— „

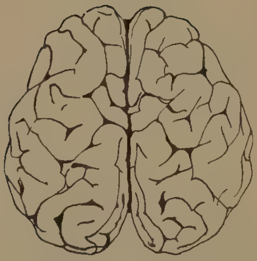

Fig. 9. Sus scrofa dom. neon. (nat. size)

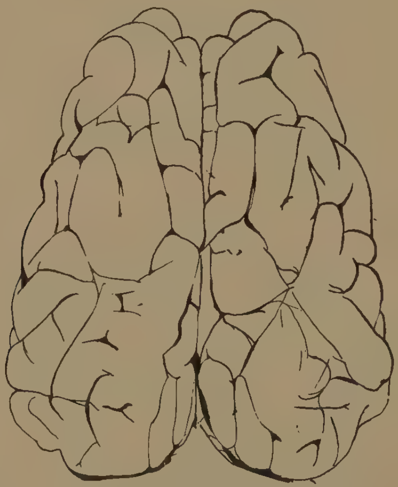

Fig. 10. Sus scrofa dom. (± nat. size) adult.

UNGULATES.

Sus scrofa neon.	3.5 cm	3.2 cm	91.4	14.— gr.
„ „ adult.	7.9 „	6.4 „	81.—	112.— „
Rusa porcinus neon.	5.4 cm	5.2 cm	96.3	54 gr.
„ „ adult.	8.2 „	7.— „	85.4	143 „
Camelop. giraf. neon.	10.2 cm	10.9 cm	108.2	466 gr.
„ „ adult.	13.5 „	11.96 „	88.6	665 „

Moreover as the extension of the whole skull (not only the endocranium) takes place especially in antero-posterior direction, we must consider the possibility that the *skull* itself in larger animals of the same family is relatively more lengthened, anteriorly because the insertion of teeth requires more place and posteriorly by the influence of the stronger cervical muscles and ligaments. On the other hand the skull narrows relatively in the larger specimens by the pressure of the larger masticatory muscles.

There is still another question that I want to discuss.

DUBOIS [1]) (l.c.) and LAPICQUE [2]) have shown that the exponent r in the formula $E = kP^r$ between adults of the same species (e.g. various dogs) is smaller ($^5/_{18}$) than in animals of different species ($^5/_9$).

DUBOIS (1922) called the first exponent "ontogenetic", the second. "phylogenetic", presuming that in larger varieties of the *same* species, as in ontogenetic (especially postembryonic) development, the increase of the brain is due merely to an *enlargement of the neurones* [3]). This neurone enlargement according to CONKLIN and G. LEVI, follows an exponent of about $^5/_{18}$ with respect to the body weight.

On the other hand, in more *cephalized* animals of various size in the larger species the enlargement of the neurones is accompanied by a *numeric increase of neurones,* the latter fact according to this anthor being responsible for the higher exponent $^5/_9$.

Is there anything in my tables that may confirm this ? Does the increase of the brain in larger varieties of the *same* species show more conformity with the enlargement of the brain during postembryonic development than it does in larger but not closely related species ?

Comparing the forebrain indices of the different varieties of the species Canis famil., we see that the difference in brainweight of only 42.3 gr. is accompanied by an increase of 18.2 units in the forebrain index.

Comparing members of the family of the Felidae (which is still very homogeneous) we find, with a maximal difference in the brain weight of 78.5 gr., a much smaller index difference, viz. 10.5. In the less homogeneous family of the Cervidae we find with a weight difference of 253 gr. an index difference below 10, (8.3), and in the still less homogeneous group of Antilopes with a still higher weight difference (359 gr.) an even smaller index difference (5.3).

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[1]) DUBOIS. Ueber die Abhängigkeit des Hirngewichts von der Körpergrösze beim Menschen. Archiv für Anthropologie Band 25, Heft 4, 1898.

[2]) LAPICQUE. Sur la relation du poids de l'encéphale au poids du corps. Comptes rendus de la Société de Biologie, Paris 1898.

LAPICQUE. Le poids encéphalique en fonction du poids corporel entre individus d'une même espèce. Bulletin de la Société d'Anthropologie de Paris, 1907.

[3]) This is true for the Norwegian rat and tame albino rat (*Sugita*) compare DONALDSON : The significance of the brainweight. Transactions of the annual meeting of the American neurological Association, Philadelphia, 1924. — Archives of Neurology and Psychiatry. March 1925, p. 385.

If now we consider the index differences during postembryonic development, we find them very large, all above 10; Lutra 15.8, Felis 12, Rusa 10.7, Sus 10.4. The lowest of these still comes in the group of the very homogeneous Felidae. All other index differences are higher and approach varieties of one species (cf. Canis familiaris).

In other words the elongation of the brain in larger animals shows a greater conformity with the ontogenetic elongation if a group is more homogeneous and its elongation resembles the ontogenetic elongation in varieties of the *same* species. This apparently is in conformity with Dubois' supposition.

That the increase of the brain in fullgrown varieties of the same species or of closely related species of one family is only based upon cell enlargement (though this is surely the most important factor) is perhaps too strongly expressed, the more so since we know that some, though not much cell division also occurs during postembryonic development (Agduhr) 1).

In regard to the question of ontogenetic brain-body-weight relations I refer to the recent investigations of Anthony and Coupin 2), on what they call the "indice de valeur cérébrale": $\frac{PE}{PE'}$, in which PE is the real brain weight of a foetus and PE' the calculated brain weight which that foetus would have *if* it were an adult of this size.

We have seen that in animals with a higher cephalization coefficient the greater brachencephaly is a consequence of the greater degree of development of the brain compared to the body, in consequence of which the brain tends to the largest volume with the smallest surface (skull). In very primitive forms, however, particularly the most primitive recent species, the forebrain is very short, and consequently brachencephaly occurs, here also, although now it is not a consequence of the greater volume, but of the shortness of the forebrain.

So Ornithorrhynchus and Echidna have an index of 111.5 and 127.6.

Among Marsupialia the primitive Caenolestes and Orolestes are conspicuous for the shortness of their forebrain, in comparison to the Opossum. In Caenolestes obscurus the forebrain index (measured after a drawing of C. J. Herrick) 3) is 125.5; in Orolestes inca (after the drawing of Miss Obenchain) 4) 137.4. Orycteropus, which is also a primitive animal, has, after a drawing of Sonntag and Woollard 5) a

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1) Agduhr. Is the postembryonic growth of the nervous system due only to an increase in number of the neurones? Proceedings Royal Acad. Amsterdam Vol. **27**, 1919.

2) Anthony et Coupin. Introduction à l'étude du développement pondéral de l'encéphale. l'Indice de valeur cérébrale au cours de l'évolution individuelle. Zagreb. 1925—1926.

3) The brain of Caenolestes. Publications of the Fieldmuseum, Zool. Vol. **14**, 1925.

4) The brains of the South American Marsupials Caenolestes and Orolestes, Publications of the Fieldmuseum, Zool. Vol, **14**, 1925.

5) A monograph on Orycteropus afer: II Nervous system, sense organs and hairs. Proceedings of the Zool. Society of London 1925. Weber counts this animal among the Ungulates.

PRIMITIVE ANIMALS COMPARED WITH NON-PRIMITIVE (—).

Animal	Forebr. length	Forebr. width	Forebr. index	Total brainweight
Monotremes.				
Ornithorrhynchus	2.6 cm	2.9 cm	111.5	11.— gr.
Echidna	4.— „	5.1 „	127.6	19.— „
Marsupials.				
Caenolestes obscurus	0.954 cm	1.2 cm	125.5	—
Orolestes inca.	0.914 „	1.24 „	137.4	—
(Macropus rob.)	5.4 „	5.— „	(92.6)	(64.— gr.)
Orycteropus	4.9 cm	5.4 cm	110.—	—
Ungulates.				
Perissodactyla				
Rhinoceros unic.	12.— „	13.3 „	111.—	864.— gr.
(Equus caballus)	10.79 „	10.17 „	(94.2)	(549.— „)
Artiodactyla				
Hippopotamus	10.3 „	11.— „	107.—	582.— „
Hippopot.; Garrod	—	—	111.6	—
(Sus scrofa dom.)	7.9 „	6.4 „	(81.—)	(138.5 „
Tragulus mem.	3.27 „	3.3 „	101.—	18.5 „
(Cervulus muntjac, small sp.)	5.23 „	4.49 „	(85.8)	(44.— „)
Sirenia.				
Halicore; Dexler [1])	6.9 cm	7.14 cm	103.5	282 gr.
Manatus; Garrod	6.9 „	7.3 „	106.—	?
Carnivores.				
Viverra civetta	4.5 cm	4.0 cm	88.—	40.5 gr.
(Paradoxurus musanga)	4.2 „	3.1 „	(73.8)	(23.5 „)

very large forebrain index, viz. 110. The same is observed in Tragulus memmina (total brainweight 18.5 gr.) [2]), which is more primitive than the Antilopes and also has a smaller cephalization coefficient and yet shows a higher index than the latter, viz. 101.

While in the above named cases the small size of these animals might be considered as a factor in the increase of brachencephaly, this is not the case with the Rhinoceros, Hippopotamus and the Sirenia. Yet Rhinoceros sumatrensis in GARROD's drawing [1]) has an index of 119. OWEN's [2]) Rhinoceros unicornis has an index of 111 with a brain weight of 864 gr. A Hippopotamus of my collection has an index of 107 [3]) with a brain weight of 582 gr. (WEBER).

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[1]) Das Hirn von Halicore dugong. Morph. Jahrb. Bnd. 45, 19.

[2]) WEBER 17.1 gr.

Halicore, calculated after the drawing of DEXLER, has an index of 103.5; Manatus, after that of GARROD [4]), 106.

Among the Viverridae, the most primitive and poorly cephalized Viverra civetta has an index of 88.8, while its relatives Mungos and Paradoxurus, though smaller, have an index of 73.5 and 76.2 respectively.

As stated above, however, the brachencephaly of the forebrain in these primitive animals has a very different character from the brachencephaly in highly cephalized animals, being the result of the shortness of the brain in the first case, whereas in highly cephalized animals it is due to greater width. The latter is a result of increase, the former, of lack of development of the forebrain.

CONCLUSIONS.

1. Higher cephalization within a certain group of animals causes brachencephaly.

2. Larger animals are usually more dolichencephalic than similarly cephalized small animals of the same group [5]).

3. The forebrain of primitive now living [6]) mammals is usually strongly brachencephalic, but this brachencephaly is of an entirely different character.

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[1]) See GARROD. Transactions of the Zool. Society of London. Vol. **10**, 1877—1879 The brain of the Sumatra Rhinoceros.

[2]) OWEN. On the Anatomy of the Indian Rhinoceros (unicornis) Ibidem vol. **4**, 1862.

[3]) GARROD's specimen had even a forebrain index of 111.6, after his drawings in the transactions of the Zool. Society of London Vol. **11**, 1885.

[4]) GARROD, Ibidem, Vol. **10**, 1877 (see also MURIE, ibidem Vol. **11**, 1879).

[5]) For the skull, this phenomenon is often stated in man. See KOHLBRUGGE: Tijdschrift v. h. Kon. Nederl. Aardrijksk. Genootschap 2de Serie. Deel **28**, 1911, p. 785.

[6]) Concerning extinct species, little may be said in this respect, because as a rule only data about the endocranium are available. But these animals are generally less cephalized (DUBOIS) than their present relatives.

Physics. — *On the change of the dielectric constant of liquid helium with the temperature. Provisional measurements.* By M. WOLFKE and W. H. KEESOM. (Comm. N⁰. 190*a* from the Physical Laboratory at Leiden.)

(Communicated at the meeting of December 17, 1927).

§ 1. *Introduction.* The dielectric constant of liquid helium has been measured at the temperature of the boiling-point under normal pressure by M. WOLFKE and H. KAMERLINGH ONNES [1]). Now it was of the greatest importance to get to know the course of the dielectric constant of liquid helium with the temperature, particularly in connection with the peculiar density-maximum, found by KAMERLINGH ONNES [2]) and more closely studied by KAMERLINGH ONNES and BOKS [3]) and which until now has not yet found an explanation.

For these measurements by which very small differences in the dielectric constant had to be accurately determined, the method, applied to former ones, was no more sufficient. For that reason a new method for measuring was developed for this purpose by one of us at the Physical Laboratory of the Technical Institute at Warsaw. This method and the corresponding arrangement are described in the following report.

§ 2. *Measuring-method. Apparatus.* The measuring-method is literally a compensation-method by which the changes of the dielectric constant in the measuring-condenser are compensated by measurable changes of the capacity of a cylinder-condenser which can be very accurately adjusted by a micrometer-screw. This cylinder-condenser we shall call in future ,,microcondenser". This microcondenser is connected parallel to the measuring-condenser which contains liquid helium. The capacity of both these condensers together has been compared with a constant capacity by means of a method depending on the beats of two high-frequent electromagnetic oscillation circuits.

The diagram of connections is represented in Fig. 1. It consists of two mutually independent oscillation circuits according to MEISSNER: 1 and 2. The oscillation circuits are fed by a joint anode-battery of 160 V in the constant-current connection [4]). In order to avoid a mutual influence of both the circuits, the high-frequency currents are arrested by both the choking-coils D_1 and D_2.

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[1]) These Proc., 27, 621, 1924. Comm. Leiden, N⁰. 171*b*.

[2]) These Proc., March 1911, p. 1093. Comm. Leiden N⁰. 119.

[3]) Reports and Communications presented by H. KAMERLINGH ONNES to the 4th International Congress of Refrigeration, London 1924. Comm. Leiden N⁰. 170*a*.

[4]) Comp. J. ZENNECK und H. RUKOP, Lehrbuch der drahtlosen Telegraphie, 1925, p. 607, fig. 547.

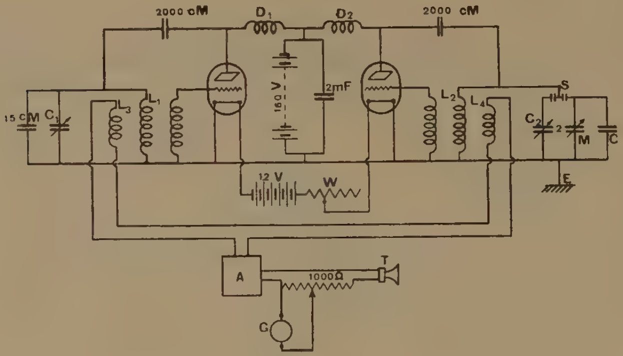

Fig. 1.

The oscillation circuit 1 consists of the self-inductions L_1 and the variable precision-condenser C_1 with a capacity of about 250 cm. Parallel to this a constant capacity of 45 cm has still been connected. In the oscillation circuit 2 is found, beside the self-induction L_2 identical to L_1, a system of condensers : the measuring condenser C which is found in the cryostat (see further), the microcondenser M connected parallel to it (fig. 2) and the normal variable condenser C_2. By changing the plugs in S either both the condensers C and M or the variable condenser C_2 can be switched in the oscillation circuit 2. Both the oscillation circuits have a joint earth-point E. For generating the oscillations the Philips-lamps type E proved to be the most suitable. The heating-current was produced by the same battery of accumulators of 12 V with in series the resistance W.

The oscillations of both the circuits 1 and 2 are superposed by means of the self-inductions L_3 and L_4. The beats caused in this way are amplified by the amplificator A. The latter consists of a detector-lamp, two low-frequency amplificator-lamps in the usual transformator-circuit and of an output-transformator. The thus amplified beats can be observed either in the radio-telephone T or in the string-galvanometer G. The string-galvanometer G has been joined to the telephone circuit in potentiometer connection. With some practice the beats can be made so slow that the swinging to and fro of the wire in the string-galvanometer can be followed.

The place of the observer is at about 4 m from the apparatus. The two variable condensers C_1 and C_2 are adjusted by the observer by means of thin cords and pulleys, the microcondenser by means of a 4 m long stick of bamboo. In this way the oscillation state during measuring is not disturbed by the movements of the observer. Further all precautions have been taken, as the earthing of the metal covers of the condensers etc. In order to avoid over-oscillations the self-inductions L_1 and L_3 on the one side and L_2 and L_4 on the other side have been coupled very loosely to the concerning regenerative-coils.

The normal variable condenser C_2, capacity about 300 cm, from the firm SPINDLER and HOYER at Göttingen has been calibrated by the P.T.R. in Berlin. This condenser has a scale divided in degrees, with nonius; the change of capacity is 1.35 cm for each degree. In order to improve the exactness of reading, a dish of 14 cm diameter with a division in degrees and nonius, was attached to the fine-regulating axis; the turning of this dish over 1 degree answers to a change of capacity to the amount of 0.17 cm.

The *microcondenser* M was made in the workshop of the Physical Laboratory I of the Technical Institute in Warsaw. Figure 2 shows this condenser in section. Within an iron cylinder A is a brass cylinder B which is isolated by an amber stop C and a fitting of ebonite C'. Exactly in the axis of this brass cylinder B is an iron cylinder D which can be exactly adjusted by means of the micrometer-screw E.

The micrometer-screw has been made very accurately and has a pitch of 0.5 mm; it possesses a head G of which the circumference is divided in 100 parts. A scale F put at one side serves for the reading of the number of complete rotations. The entire change of capacity of the microcondenser by screwing in the cylinder D was accurately measured in the arrangement itself by means of the normal variable condenser described above; the mean of 10 different measurings was 5.2 cm. This agrees up to 1 % with the value which was calculated from the measurements of the condenser. At the zero-point in the beginning the movable cylinder D is already 10 mm in the isolated cylinder B, so that the boundary disturbances of the electric field do not come into account. In the final state the cylinder D is only 40 mm in the cylinder B, so that it is still 20 mm removed from the bottom. Through this the proportionality between the change of capacity and the number of rotations of the micrometer-screw E is amply secured. To the change of capacity of 5.2 cm correspond 60 rotations of the micrometer-screw each of 100 divisions at a rotation, so 6000 divisions in all, hence to each division of the head G of the micrometer-screw a change of capacity corresponds of $8.7 \cdot 10^{-4}$.

The cryostat with the measuring-condenser agrees substantially with that one, which has been used with the first measurings of the dielectric constant of liquid helium [1]), and has been represented in Fig. 3. The condenser now consists of 6 concentric brass cylinders with a capacity of about 175 cm. The movable contact, which was formerly found at K, has been replaced by a junction directly soldered on.

Further above the glass vessel g, silvered over inside, in which is the measuring condenser C, a reservoir A was placed which was also filled with helium in order to keep the surface of helium, notwithstanding the volume-contraction at the cooling, continually above the upper part of the condenser-vessel g. The condenser C itself hangs by the leads.

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[1]) These Proc. 27, p. 622; Comm. Leiden N⁰. 171*b*. p. 11, fig. 3.

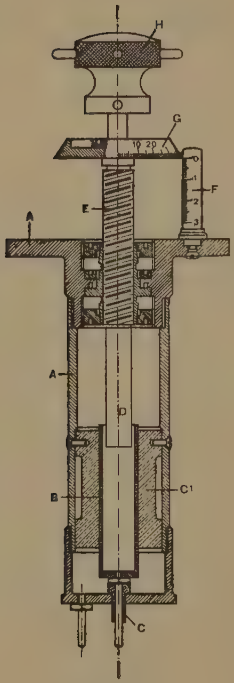

Fig. 2.

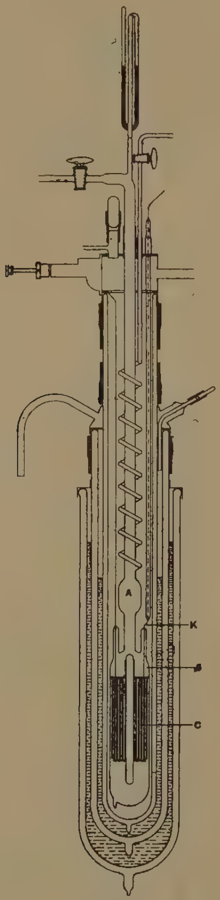

Fig. 3.

The *measurements* were made in the following way. As has already been said, they depend on the comparison of the unknown capacity with that of a normal-condenser, the small changes of the unknown capacity being compensated by means of the micro-condenser. The course of a measuring is as follows : In the oscillation circuit 2 (Fig. 1) the measuring-condensor C with the microcondensor M is switched in ; then by turning the condensor C_1 in the oscillation circuit 1 the beats are made slower till the wire in the string-galvanometer G remains quiet; this point is accurately adjusted. Then without changing the condenser C_1 in the oscillation circuit 1 the normal-condenser C_2 is connected with the oscillation circuit 2 by means of the plug S and regulated so long until the wire of the string-galvanometer remains quiet again. With this, attention should be paid that the adjusting is always brought about through a last turning in the same direction as well by the condenser C_1 as also by C_2. The capacity of the two condensers C and M is then equal to that of the normal-condenser C_2. However with this the capacities of the leads have not been reckoned with, so that this method only allows to measure capacity-differences, not the absolute values of the capacities. Small changes of the capacity of the measuring-condenser C are measured by compensating with constant adjusting of the normal-condenser C_2 at a definite initial-capacity of the measuring-condenser, each capacity-change of the latter by means of the microcondenser M and then to read scale and head from this.

The *accuracy of adjusting the normal-condenser* was determined in this way that at a constant capacity of the measuring-condenser and a constant position of the microcondenser the normal-condenser was adjusted several times and the position of the dish was read. From that followed as mean error of adjusting of the normal-condenser a value which corresponds to a capacity of ± 0.03 cm.

In suchlike way the *adjusting accuracy of the microcondenser M* was also determined. The mean deviation was ± 6 divisions of the head which corresponds to a capacity of about $\pm 5.10^{-3}$ cm. With a capacity of about 175 cm of the measuring condenser this adjusting-accuracy allows a capacity-change of about 3.10^{-3} % still to be measured, so that the accuracy of the fourth decimal in the relative value of the dielectric constant is completely secured.

At all measurements the wave-length was 600 m. Before the beginning of a measurement some time was waited after the putting into work of the apparatus, until the oscillation circuits were stable. This was the case after about an hour.

§ 3. *Capacity of the measuring-condenser.* As has already been said the here described measuring-method was worked out specially for the accurate measurement of very small capacity-changes and it is not well fit for the accurate determining of the absolute values of capacities. For that reason an indirect method was applied for measuring the absolute value of the capacity of the measuring-condenser. With this method the capacity was determined from the difference of the capacities in the vacuum and in liquid helium boiling under atmospheric pressure and from the formerly (Comm. Leiden N⁰. 171*b*) measured value of the dielectric constant of liquid helium at the boiling-point.

If we call C_0 the capacity of the measuring-condenser in a vacuum at the normal boiling-point of helium, 4.21° K., C the capacity in liquid helium at the same temperature, K_0 the dielectric constant of liquid helium at that temperature, $\triangle C$ the difference of the two capacities, measured and read on the normal condenser, then we see that :

$$\triangle C = C - C_0, \text{ and } C = KC_0, \text{ with } K_0 = 1.048,$$

from which follows :

$$C_0 = \frac{\triangle C}{K_0 - 1} \quad \dots\dots\dots\dots \quad (1)$$

First the measurement was made in a vacuum. The glass vessel g in the cryostat (Fig. 3) which contains the measuring-condenser, was evacuated and filled with helium-gas under a pressure of some mm of mercury. The cryostat was filled with liquid helium, so that the vessel g was quite immersed. The gasfilling in g has no measurable influence on the value of the capacity ; its purpose was to secure the exchange of temperature between the liquid helium in the cryostat and the measuring-condenser. The temperature in the cryostat was deduced from the vapour-pressure of the helium by means of the formula given by KAMERLINGH ONNES and WEBER 1).

We found $\triangle C = 8.36$ cm from which by means of (1) follows for the capacity of the measuring-condenser empty at the boiling-point of helium :

$$C_0 = 174 \text{ cm.}$$

The formerly measured value of the dielectric constant of liquid helium at the boiling-point (see Comm. Leiden N⁰. 171*b*) is exact up to 0.1 %. Accordingly K_0—1 is exact up to about 2 %. Since the readings on the normal-condenser are much more exact we can consider the value of C_0 exact as up to 2 %. The accuracy of this measurement is of secondary importance to the proper purpose of the research, treated in this Communication, since the relative values are only required in investigating the variation of the dielectric constant of the liquid helium with the temperature.

§ 4. *Change of the dielectric constant of liquid helium with the temperature.* For these measurements the glassvessel with the measuring-condenser in the cryostat was filled with liquid helium, while it was immersed itself in liquid helium.

The *temperature* in the cryostat was adjusted in the usual way by reducing the vapour-pressure of the helium in the cryostat till the point desired was reached; its value was deduced from the vapour-pressure (see § 3). As initial temperature served the normal boiling-point of helium.

The changes of the dielectric constant at different temperatures were measured as follows. The normal-condenser was accurately adjusted at the initial-temperature of the liquid helium and then changed no more during the entire measurement. At each further temperature the adjusting was only

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1) These Proc., Sept. 1915, p. 493. Comm. Leiden N⁰. 147*b*.

performed with the aid of the microcondenser. The changes of capacity were then read on the latter.

From the position of the microcondenser the value of the dielectric constant at a definite temperature can be calculated in the following way: If we call C the capacity of the measuring-condenser filled with liquid helium at the initial temperature and if $\triangle C$ be the increase of this capacity by the passing on to the new temperature, then the dielectric constant K at this new temperature follows from :

$$K=\frac{C+\Delta C}{C_0}=K_0\left(1+\frac{\Delta C}{C}\right), \quad \ldots \ldots \quad (2)$$

where $C = K_0 C_0 = 182.4$.

With that the capacity C_0 of the empty condenser has been accepted as constant for all temperatures in liquid helium. This may be done without objection as the relative capacity-changes in consequence of the thermal expansion in this temperature-region should make themselves perceivable only in the 6th decimal of the dielectric constant.

The first measurements were made on the 11th of June 1927. The results obtained are given in table I. The 4th column contains the change in the position of the microcondenser counted from that one at the initial temperature, the 5th the relative value of the dielectric constant calculated according to (2), the 6th that one of the dielectric constant itself, calculated from the value formerly measured at the normal boiling-point of helium.

TABLE I.

N^0.	p mm Hg	T $^\circ K$.	Micro-condenser.	$\frac{K}{K_0}$.	K
1	766.5	4.21	0	1	1.0480
2	82.9	2.64	1735	1.00824	1.0566
3	69.6	2.55	1920	1.00912	1.0576
4	60.1	2.48	1988	1.00944	1.0579
5	49.8	2.39	2054	1.00976	1.0582
6	38.1	2.28	2211	1.01050	1.0590
7	25.8	2.12	1948	1.00925	1.0577
8	13.8	1.90	2013	1.00956	1.0580

With the values of K from table I and the densities of the liquid helium according to KAMERLINGH ONNES and BOKS [1]) the molecular electric polarization can be calculated by means of the formula of CLAUSIUS—MOSOTTI. These values are united in table II.

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[1]) H. KAMERLINGH ONNES and J. D. A. BOKS, l. c., p. 1 note 3.

After attempts of a repetition of this measurements had failed on the 17th of June and on the 12th of July, both times through the breaking of the helium-glass in the cryostat, a second series of measurements was made on the 19th of July. The obtained results are given in table III.

TABLE II.

T °K.	ρ	$\frac{K-1}{K+2}\cdot\frac{1}{\rho}$
4.21	0.1251	0.1259
2.64	0.1443	0.1283
2.55	0.1451	0.1299
2.48	0.1454	0.1304
2.39	0.1459	0.1305
2.28	0.1462	0.1320
2.12	0.1458	0.1295
1.90	0.1455	0.1305

TABLE III.

N⁰.	Time	p mm Hg	T °K.	Micro-condenser.	$\frac{K}{K_0}$	K
1	14^{u} 26^{m}	753.0	4.19	0	1.00000	1.0480
2	15 25	48.8	2.39	1933	1.00922	1.0577
3	39	40.1	2.30	2024	1.00966	1.0581
4	59	34.1	2.24	2014	1.00961	1.0581
5	16 09	30.2	2.19	1975	1.00942	1.0579
6	22	24.9	2.11	1937	1.00924	1.0577
7	34	19.9	2.03	1974	1.00942	1.0579
8	46	10.1	1.80	1921	1.00917	1.0576
9	58	8.8	1.75	1898	1.00906	1.0575
10	17 11	31.1	2.20	2027	1.00967	1.0581
11	19	36.2	2.26	2041	1.00974	1.0582 ¹)

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¹) During this measurement the helium in the cryostat had sunk so low, that the temperature of the condenser began to become uncertain.

During these measurements the circumstances were less favourable than with the preceeding series. Besides that we apparently did not wait long enough until the temperature should have completely regulated itself, a possible source of errors had also arisen because the leads outside the cryostat were possibly in connection with the earth through a precipitated moisture, through which an undefined most probably also variable resistance was switched parallel to the measuring-condenser.

As it appears from the graph (Fig. 4) the two measuring-series point to a discontinuity, a jump, in the course of the dielectric constant with the temperature.

Since this result however principally depends only on one series of measurements (the first) we decided to wait for a new series of measurements under improved circumstances before passing on to publication. However, now that in the mean time in different ways (compare the following communication) the appearance of an abrupt change in the liquid helium has been asserted we were of opinion not to put off the publication any longer. Meanwhile we consider the results given here as being only provisional, also because there is some possible doubt about the accurate equalness of temperature of the helium in the condenser with that of the helium in the cryostat.

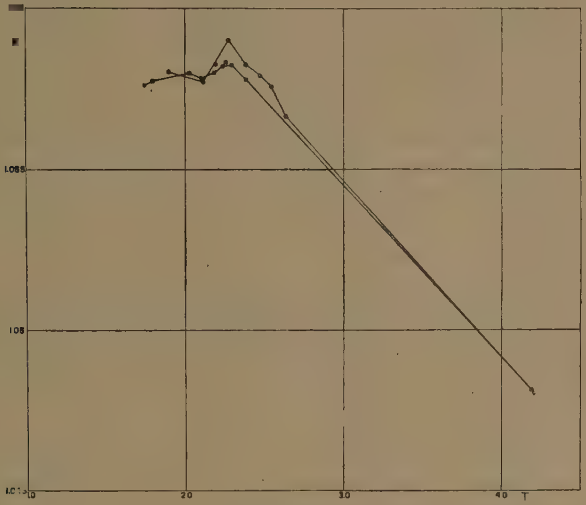

Fig. 4.

⊙ Measurements of June 11, 1927.

⊡ Measurements of July 19, 1927.

Physics. — *Two different liquid states of helium.* By W. H. KEESOM and M. WOLFKE. (Communication N⁰. 190*b* from the Physical Laboratory at Leiden.)

(Communicated at the meeting of December 17, 1927).

§ 1. When measuring the dielectric constant of liquid helium between the boiling-point and 1.9° K. on June 11th last, we observed [1]) that at a temperature almost corresponding with that one at which KAMERLINGH ONNES had found a maximum in the density curve, the dielectric constant shows a sudden jump or at least a jump made in a very small temperature-region. The thought suggested itself that at that temperature the liquid helium transforms into an other phase, liquid as well. If we call the liquid, stable at the higher temperatures "liquid helium I", the liquid, stable at the lower temperatures "liquid helium II", then the dielectric constant of liquid helium I should be greater than that of liquid helium II. Seeing that this result was only founded on one series of measurements we wished to repeat the measurements in order to get more security. Two attempts for that purpose failed (on June 17th and July 12th). At a repetition on July 19th the circumstances were less favourable, so that in our opinion the results, though they *did* point in the same direction as those of June 11th, did not yet give a sufficient affirmation. So we decided to wait for new measurements.

§ 2. Meanwhile our attention had been drawn by the following facts:

a. The results of the density-measurements of KAMERLINGH ONNES and BOKS [2]) may be associated as well, if not better (see Fig. 1, which we must compare with Fig. 5 of Comm. No. 170*b*) with the admission of a jump in the density at 38 mm helium-pressure (to the amount of about 1⁰/₀₀, the density of liquid helium II smaller than that of liquid helium I) than with that of a smooth maximum at that place.

b. In the paper of DANA and KAMERLINGH ONNES [3]) on the specific heat of liquid helium results are only mentioned at temperatures higher than the one expressed above. However, they have also experimented round that temperature, but then they found values, which did not agree with those measured at higher temperatures. For a part of the experiments this will be due to condensation of the helium-vapour, as DANA and KAMERLINGH ONNES point out. With some experiments leading to deviating results a suchlike condensation would not have been expected according to the data, which are kept in the archives of the laboratory.

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[1]) These Proc., p. 81. Comm. Leiden N⁰. 190*a*.

[2]) Comm. Leiden N⁰. 170*b*.

[3]) These Proc. **29**, 1061, 1926. Comm, Leiden N⁰. 179*d*.

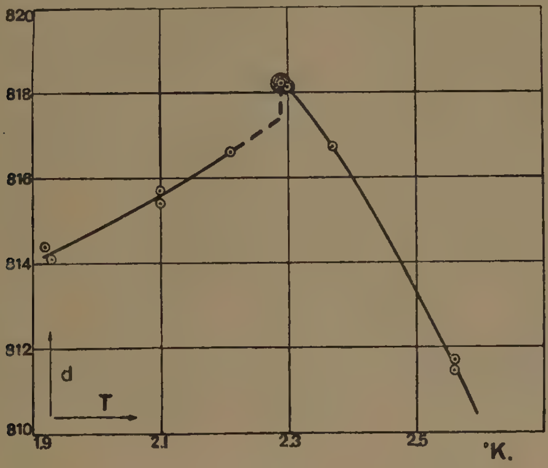

Fig. 1.

In fact from three of those experiments we could deduce nearly corresponding values for the transformation heat with a mean value:

liquid helium I = liquid helium II + 0.13 cal./gram.

c. The results of the measurements of DANA and KAMERLINGH ONNES [1]) concerning the heat of vaporization of liquid helium clearly point to a jump in the heat of vaporization (see Fig. 3 there). They already say that it may be possible that those results should indicate "that near the maximum density something happens to the helium, which within a small temperature range takes place perhaps even discontinuously" [2]). According to those results at the transformation point the heat of vaporization of liquid helium II would be greater than that of liquid helium I and that to an amount which corresponds in order of magnitude with the value of the transformation heat given under *b*.

d. Measurements of VAN URK, KEESOM and KAMERLINGH ONNES [3]) concerning the surface tension of helium seem to indicate clearly a jump in the value of the surface tension between 33 and 39 mm mercury-pressure to the amount of about 3 % (see Fig. 2 there) and in such a way that the surface tension of liquid helium II is smaller than that of liquid helium I.

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[1]) These Proc. 29, 1051, 1926. Comm. Leiden N⁰. 179*c*.

[2]) These Proc. 29, 1057 note 2, 1926. Comm. Leiden N⁰. 179*c*, p. 31 note 1. The authors continue: "The change of density of the liquid also indicates something of the same kind".

[3]) These Proc. 28, 958, 1925. Comm. Leiden N⁰. 179*a*.

§ 3. While the repetition, under improved circumstances, of the measurements of the dielectric constant mentioned in § 1, had to be put off on account of the absence of one of us from Leiden [1]), the provisional value of the transformation-heat mentioned in § 2 *b* raised the surmise that the transformation of the liquid helium should show itself in the cooling- or heating-curve, when a quantity of liquid helium was being cooled regularly (by gradual lowering of the pressure) resp. heated itself in consequence of the supply of heat by conduction and radiation.

This experiment was carried out on November 18th last. The course of the pressure on a mercury-manometer was thereby followed with the eye. With the cooling as well as with the heating a short halt of the manometer was stated each time at 39 mm pressure.

Besides a stirrer, with which was continually stirred during the experiment, the helium-bath contained a constantin resistance. The intention was to register photographically by means of this the course of the temperature with the time. This, however, did not succeed on that day partly because the heating just like the cooling passed off sooner than we had expected, partly because the influence of a change in temperature of the leading wires to the constantin-resistance (Diesselhorst-galvanometer, Thomsonbridge connection) did not appear to be sufficiently eliminated.

A repetition took place on Nov. 29th. Seven experiments were made: 4 coolings and 3 heatings.

The course of the mercury-manometer showed again the same phenomenon as on Nov. 18th. Especially the course by heating was very characteristic. The manometer beginning from about 5 mm at first rose comparatively slowly, then stood still for a moment (always at the same point) and after that suddenly began to rise quickly. At one of these experiments, at which the manometer stood still (it was estimated during 4 sec.) the pressure was read 38 mm.

During each of those experiments the deflection of the galvanometer in the Thomsonbridge connection could be registrated [2]). On each of the thus obtained resistance-time-registrations, which can be almost considered as temperature-time-curves, the transformation-point can be clearly recognized. Visually nothing was to be seen of the forming of a second phase in the liquid helium. This need not astonish us as the refraction-indices of the two phases will differ very little [3]).

From the fact that at lower temperatures the liquid is evidently being moved as easily by the stirrer as at higher it also appears that the viscosity of the helium II must be small. On December 7th measurements were made concerning the resistance of the constantin wire mentioned above, particularly near the transformation-point, the pressure of the helium being kept constant during each measurement. We shall possibly revert to these experiments later. After that we had the opportunity to register still 3 cooling- resp. heating-curves in the way mentioned above. During these registrations the temperature changed considerably slowlier than at the preceeding experiments. Though the then obtained curves show peculiarities, which we cannot yet quite interpret, still each of them shows very clearly that at a pressure of about 38 mm something very peculiar takes place.

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[1]) We hope that this repetition will soon take place.

[2]) We heartily thank Mr. J. N. VAN DEN ENDE, phil. cand., for his assistance in these experiments.

[3]) Herewith should be compared what has been observed at the experiments concerning the solidification of helium: these Proc. 29, 1136, 1926, Comm. Leiden N⁰. 184*b*.

On December 16th measurements were made by Dr. SOPHUS WEBER, Mr. NØRGAARD and one of us on the vapour-pressure of helium particularly also near the point in question here. This time the temperature was measured with a heliumthermometer of which the pressure was read by means of a hot-wire manometer. Later on the results of these measurements will be communicated. Fig. 2 gives the successive readings of the galvanometer in the hot-wire manometer connection, when after a series of these measurements the heliumbath heated itself. The transformation-point is very clearly indicated in this heating-curve.

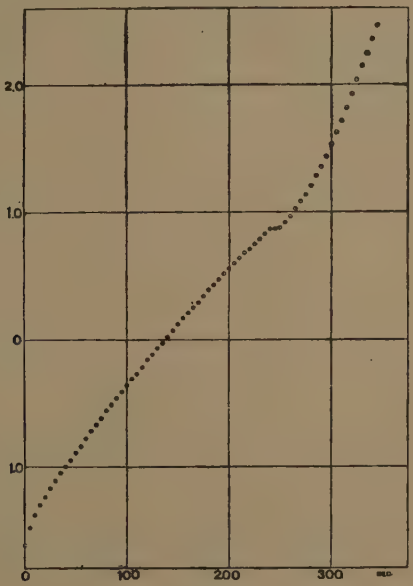

Fig. 2.

§ 4. Through the preceeding experiments we think the fact to be established that at a presure of about 38 mm a very peculiar change appears in the liquid helium, which in any case takes place within a very small temperature-region (0.08 of a degree). Though we have not been able to observe visually the two phases side by side, we yet think it most probable that we have to do here with two different states of liquid helium, which transform into each other at the temperature corresponding to the pressure mentioned. Of those two phases the liquid helium II (stable at the lower temperatures) compared with liquid helium I has:

a smaller density,
a greater heat of vaporization,
a smaller surface-tension,

while the transformation liquid helium II → liquid helium I takes place with an absorption of heat, of which the amount can be valued for the present àt 0.13 cal./gram.

We put the temperature, at which the transformation takes place, for the present at 2.3° K. [1]).

It is remarkable that this transition takes place at a temperature, which, roughly speaking, may be considered to correspond with those at which other substances have their melting-point.

So helium has a triplepoint liquid I-liquid II-vapour, which until now has only been found with a number of complicately composed substances, which have a mesomorphic (liquid crystalline) phase. Whether the latter is also the case with helium, will have to appear from further experiments.

Likewise whether and, if so, in which point the transformation curve liquid I-liquid II meets the melting-curve.

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[1]) It is evident that the deduction of the temperature from the value of the vapour-pressure especially near this point, shall want a reconsideration.

Botany. — *Notes on Pteridophyta from Djambi, Sumatra.* By O. POSTHUMUS. (Communicated by Prof. J. C. SCHOUTE.)

(Communicated at the meeting of January 28, 1928).

The following remarks are based on the Pteridophyta of a collection, which I made in the interior of the residency of Djambi (Sumatra), from July to November 1925. The plants are partly collected near Bangko, situated at the junction of the S. Mesoemai with the S. Merangin, where the latter ceases to be navigeable; partly in the surroundings of the camp Selemboekoe, about 30 K.M. west of Bangko; only one specimen near Sarolangoen and two near Paoe, both situated at the S. Tembesi.

The landscape can be described as a plateau, with but a slight relief, in which, in consequense of a recent upheavel, the rivers have made deep ravines; palaeozoic strata, which are covered by tertiary and quaternary deposits of vulcanic origin, come to the surface in the deepest parts.

Originally this region was covered with primary forest, continuous with that of the Barisan mountains, which are situated more westward. A considerable portion, however, in the more accessible parts, has been destroyed by the natives for their "ladangs" (not irrigated rice fields). These are left to themselves after one or two harvests have been obtained; then secundary jungle appears, or, especially during the last years, rubber plantations are planted by the natives.

The climate is aequatorial; the rainfall is rather high and almost equally distributed throughout the year. No well pronounced dry season occurs and the humidity of the air, especially in the shadow of the primary forest, is high and does not seem to be liable to much variation. This is shown in the fern-vegetation of the primary forest by the presence of Hymenophyllaceae, which are found on trees, but also on the soil, on steep, shadowy slopes, or on bigger boulders in the bed of the smaller rivers; they were never found in the secundary vegetation.

The high degree of humidity is especially favourable to reboisement. Some months after the rice-fields have been left to themselves, the young trees (bloekar) already attain a considerable height. Alang-alang (Imperata cylindrica Beauv.) is found only on the roads, on some clearings and in older native rubber plantations, apparently only if the vegetation is periodically burnt down. If left to itself, the (secundary) forest will come up soon.

In the following list the Ferns are arranged after the system of the Index Filicum of C. CHRISTENSEN, whose nomenclature is also followed;

the distribution of the species is mentioned as far as known to me, from the literature, which is quoted only in some cases in the discussion, and from material of the Buitenzorg herbarium.

HYMENOPHYLLACEAE.

Trichomanes bipunctatum Poiret, Encycl., VIII, p. 69, 1808.

In primary forest, covering the rocks, on shadowy humid places ; near S. Merangin, opposite to M. Djangkang, about 170 M. above sea-level : N⁰. 618, 27 VII, 1925.

Tropical Asia and Africa, Polynesia.

Trichomanes hispidulum Mettenius ; Kuhn, Linnaea, XXXV, p. 389, 1868.

In primary forest, on the soil, in humid shadowy places ; near camp Selemboekoe, about 180 M. above sea-level : N⁰. 613, 26 VII, 1925.

Malacca, Borneo.

Trichomanes humile Forster, Prodrome, p. 84, 1780.

In primary forest, covering steep, humid, shadowy, rocky slopes ; near S. Merangin, opposite to M. Djangkang, about 140 M. above sea-level : N⁰. 625, 27 VII, 1925.

Malay Archipelago, Formosa, N. Zealand, Polynesia.

Trichomanes singaporianum (v. d. Bosch) van Alderwerelt van Rosenburgh, Bull. Jard. Bot. de Buitenzorg, (2) XX, p. 25, 1915.

In primary forest, on moist shadowy places, on the soil or on the base of old trees ; near the camp Selemboekoe about 180 M. above sea-level : N⁰. 612, 26 VII, 1925.

Malacca, Mergui islands.

Trichomanes sumatranum van Alderwerelt van Rosenburgh, Bull. dept. agric. Indes. néerl., XVIII, p. 4, 1908.

In primary forest, in shadowy, humid, places, frequent on the boulders in small rivers ; M. Karing, about 140 M. above sea-level : N⁰. 604, 25 VII, 1925 ; S. Karing, about 140 M. above sea-level : N⁰. 764, 24 VIII, 1925.

Sumatra, Java.

Hymenophyllum holochilum (v. d. Bosch) C. Christensen, Index Filicum, p. 36, 1905.

In the primary forest, on old trees : S. Selemboekoe, near the camp, about 180 M. above sea-level : N⁰. 785, 25 VIII, 1925.

Sumatra, Malacca, Banca, Riouw, Lingga-Archipelago, Java, Borneo. Philippines, New Guinea.

Hymenophyllum subrotundum van Alderwerelt van Rosenburgh, Bull. Jardin Bot. de Buitenzorg (2) XX, p. 19, 1915.

In primary forest, epiphytic, in humid, shadowy places, forming tufts (together with Polypodium inconspicuum Bl.) ; S. Selemboekoe, near the camp, about 180 M. above sea-level : N⁰. 786, 25 VIII, 1925 ; S. Mengkarang, about 200 M. above sea-level : N⁰. 1082 bis, IX, 1925.

Sumatra.

CYATHEACEAE.

Cibotium barometz J. Smith, London Journal of Botany, I, p. 437, 1842.

Rather short three-fern ; in the primary forest on steep riverbanks ; S. Merangin opposite to M. Djangkang, about 140 M. above sea-level : N⁰. 751, 21 VIII, 1925.

China, Hongkong, Formosa, Sumatra, Banca, Malacca, Borneo, Moluccas.

Cyathea moluccana R. Brown ; Desvaux, Prodrome, p. 322, 1827.

Small tree-fern, about ½ M. high ; leaves about 3 M. long ; occurs scattered in secundary forest (about 12 years old) ; near Bangko, about 60 M. above sea-level : N⁰. 481, 10 VII, 1925.

Sumatra, Malacca, Riouw, Lingga-Archipelago, Borneo, Moluccas.

Alsophila latebrosa Wallich ; Presl, Tent. Pterid, p. 62, 1836.

In native rubber plantations, which are about 8 years old ; near Bangko, about 60 M. above sea-level : N⁰. 568, 18 VII, 1925.

British India, Malay Archipelago.

DIPTERIDACEAE.

Dipteris conjugata Reinwardt, Sylloge plantarum II, p. 3, 1824.

In primary forest, on steep stony slopes of the river, in shadowy or rather open places ; S. Mengkarang between doesoen Baroe and M. Koekoen, about 200 M. sea-level : N⁰. 892, 19 IX, 1925.

Malacca, Malay islands, Polynesia.

POLYPODIACEAE.

Dryopteris calcarata (Blume) O. Kuntze, Revisio generum plant., II, p. 812, 1891.

In primary forest, on and between the boulders in the shadowy bed of small rivers ; S. Karing, about 140 M. above sea-level : N⁰. 692, 15 VIII, 1925.

Br. India, South China, Malay Archipelago, Polynesia.

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Dryopteris Dayi (Beddome) C. Christensen, Index Filicum, p. 260, 1905.
In primary forest, on the soil, in rather shadowy spots; near S. Ketidoeran Siamang, about 160 M. above sea-level: N⁰. 908, 15 X, 1925.
Malacca.

Dryopteris didymosora (Parish) C. Christensen, Index Filicum, p. 262, 1905.
In native rubberplantations (about 10 years old), in which the undergrowth is now and then destroyed; scattered, in rather open places; near the camp Selemboekoe, about 180 M. above sea-level: N⁰. 736, 19 VIII, 1925.
Northern India, Assam, Malacca, Borneo, Amboina, China.

Dryopteris malayensis C. Christensen, Monogr. Dryopteris, I, p. 171, 1913.
In primary forest, on the soil, in shadowy places; near doesoen Baroe (S. Merangin), about 200 M. above sea-level: N⁰. 855, 4 IX, 1925
Malacca, Malay islands, Philippines.

Dryopteris salicifolia (Wallich) C. Christensen, Index Filicum, p. 290, 1905.
In primary forest, growing on rocky slopes on rather bright places near the river; M. Karing, about 140 M. above sea-level: N⁰. 655, 3 VIII, 1925.
Sumatra, Malacca, Borneo.

Dryopteris sarawakensis (Baker) van Alderwerelt van Rosenburgh, Malayan Ferns, p. 200, 1909.
In primary forest, in the shadow on steep riverbanks, and between the boulders in the river-bed; M. Karing, about 140 M. above sea-level: N⁰. 602, 25 VII, 1925.
Borneo.

Dryopteris truncata (Poiret) O. Kuntze, Rev. gen. Plant., II, p. 814, 1891.
In secundary forest, on rather moist, shadowy places; near Bangko, about 60 M. above sea-level: N⁰. 475, 10 VII, 1925.
Br. India, Malay Archipelago, Trop. Australia, Polynesia, Madagascar.

Dryopteris unita (Linn.) O. Kuntze, Rev. gen. Plant. II, p. 811, 1891.
Along roads, on open fields and in older native rubberplantations, in which the undergrowth is destroyed now and then; along the road from Limboer to Bangko, about 50 M. above sea-level: N⁰. 543. 17 VII, 1925.
Br. India, Malay Archipelago, Polynesia, Madagascar.

Dryopteris urophylla (Wallich) C. Christensen, Index Filicum, p. 299, 1905.

In native rubberplantations, scattered, on rather humid places; near Bangko, about 60 M. above sea-level: N⁰. 474, 10 VII, 1925.

Br. India, Malay Islands, Polynesia, Madagascar.

Dryopteris verruculosa van Alderwerelt van Rosenburgh, Bulletin Jardin Bot. de Buitenzorg (2) X, p. 12, 1913.

In primary forest; near doesoen Baroe (S. Merangin), about 200 M. above sea-level: N⁰. 856, IX, 1925.

Eastern-Java, Sumatra.

Didymochlaena truncatula (Swartz) J. Smith, Journal of Botany, IV, p. 169, 1841.

In primary forest; near Soengai Manau, about 400 M. above sea-level: N⁰. 946, X, 1925.

All tropical countries.

Aspidium angulatum (Willdenow) J. Smith in Mettenius, Ann. Lugd. Bat., I., p. 239, 1864.

In primary forest; scattered on shadowy spots; near road to S. Manau, between S. Karing and S. Selemboekoe, about 180 M. above sea-level: N⁰. 845, 31 VIII, 1925.

Malacca, Borneo, Moluccas, New Guinea.

Aspidium nebulosum (Baker) C. Christensen, Index Filicum, p. 84, 1905.

In primary forest; near doesoen Baroe (S. Merangin) about 200 M. above sea-level: N⁰. 851, IX, 1925.

Sumatra, Banca.

Aspidium singaporianum Wallich; Hooker et Greville, Icones Filicum, pl. 26, 1827.

In primary forest; scattered, on shadowy spots; S. Merangin, opposite to M. Djangkang; about 140 M. above sea-level: N⁰639, 27 VII, 1925.

Sumatra, Banca, Malacca, Borneo.

Aspidium vastum Blume, Enumeratio plant. Jav., p. 142, 1828.

In primary forest, scattered on shadowy spots; near doesoen Baroe (S. Merangin); about 200 M. above sea-level: N⁰. 853, IX, 1925.

Br. India, Birma, Malay Archipelago.

Polybotrya appendiculata (Willdenow) J. Smith, Journal of Botany, IV, p. 150, 1841.

In primary forest, at shadowy, humid, steep, rocky slopes; M. Karing. about 140 M. above sea-level: N⁰. 606, 25 VII, 1925.

Tropical Asia.

Nephrolepis cordifolia (Linnaeus) Presl, Tentamen Pteridographiae, p. 179, 1836.

In primary forest, hanging down at steep, rather exposed, rocky slopes; S. Merangin opposite to M. Djangkang, about 140 M. above sea-level: N⁰. 623, 27 VII, 1925.

Tropical countries.

Humata angustata J. Smith, Journal of Botany, III, p. 416, 1841.

In primary forest; with a long rhizome creeping on trees on rather dry spots; near M. Karing, about 160 M. above sea-level: N⁰. 895, 6 IX, 1925.

Sumatra, Banca, Riouw, Lingga-Archipelago, Malacca, Borneo.

Humata heterophylla (Smith) Desvaux, Prodrome, p. 323, 1825.

In primary forest; creeping with a long rhizome on trees, scattered on rather dry places; between S. Karing and S. Selemboekoe, about 180 M. above sea-level: N⁰. 844, 31 VII, 1925.

Malay Archipelago.

Humata repens (L. Fils) Diels, in Engler-Prantl, Nat. Pflanzenfam., I, abt. IV, p. 209, 1899; var. *minor* Nees.

In primary forest, scattered, on trees; near S. Ketidoeran Siamang, about 140 M. above sea-level: N⁰. 761, 24 VIII, 1925.

S. China, Malay Archipelago, trop. Australia.

Davallia triphylla Hooker, Species Filicum, I, p. 162, pl. 46 A, 1846.

In higher parts of the primary forest, climbing, with a long rhizome in groups; near the road to Soengai Manau on the watershed between S. Karing and S. Selemboekoe, about 180 M. above sea-level: N⁰. 843, 31 VIII, 1925.

Malacca.

Tapeinidium gracile van Alderwerelt van Rosenburgh, Malayan Ferns, p. 315, 1909.

In primary forest, on humid shadowy slopes; near the camp Selemboekoe, about 180 M. above sea-level: N⁰. 778, 25 VIII, 1925.

Sumatra, Borneo, Java, Philippines, New Guinea.

Schizoloma ensifolium (Swartz) J. Smith, Journal of Botany, III, p. 414, 1841.

In native rubber-plantations, which are about 7 years old and have but little undergrowth, scattered; near Bangko, about 60 M. above sea-level: N⁰. 461, 9 VII, 1925.

Tropical Africa, Asia and Australia; Polynesia.

Lindsaya decomposita Willdenow, Species plantarum, V., p. 425, 1810. (L. davallioides Blume, Enumeratio plant. Jav. p. 218, 1828.)

In primary forest, in shadowy places, on the soil or on the base of old trees; between the camp Selemboekoe and S. Karing, about 180 M. above sea-level: N⁰. 825, 29 VIII, 1925; S. Merangin, opposite to M. Karing; N⁰. 680, 9 VII, 1925.

Trop. Asia, Australia, Polynesia.

Lindsaya lancea (Linnaeus) Beddome, Ferns Brit. Ind., Suppl. p. 6, 1876.

In primary forest, on the soil or on the base of old trees near the camp Selemboekoe, about 180 M. above sea-level: N⁰. 616, 26 VII, 1925.

Trop. America, Ceylon, Malacca, Sumatra, Java, Borneo.

Lindsaya scandens Hooker, Species Filicum, 1, p. 205, pl. 63 B, 1846.

In primary forest, in humid, shadowy places, on the soil or on the base of old trees; near the road to S. Manau, about 200 M. above sea-level: N⁰. 668, 5 VIII, 1925.

Malay Archipelago.

Diplazium aequibasale (Baker) C. Christensen, Index Filicum, p. 227, 1905.

In primary forest, on shadowy, moist places; near creeks, between the rocks; S. Karing, about 140 M. above sea-level: N⁰. 695, 15 VIII, 1925; S. Karing, near the S. Merangin: N⁰. 607, 25 VII, 1925.

Borneo.

Diplazium bantamense Blume, Enumeratio, p. 191, 1828.

In primary forest, in shadow, on the soil; near the S. Merangin, opposite to M. Djangkang, about 140 M. above sea-level: N⁰. 638, 27 VII, 1925.

Tropical Asia, Japan.

Diplazium confertum (Baker) C. Christensen, Index Filicum, p. 230, 1925.

In primary forest, on the soil, sometimes at steep, shadowy banks of the creeks; S. Selemboekoe, about 150 M. above sea-level: N⁰. 783, 25 VIII, 1925; S. Boekit Tinggi, near doesoen Baroe, about 140 M. above sea-level: N⁰. 664, 4 VIII, 1925.

Sumatra, Borneo, Celebes.

Asplenium cymbifolium Christ, Bulletin Herbier Boissier (2) VII, p. 999, 1906.

In primary forest in shadowy places; the rhizome vertically climbing with short internodes, leaves with broad bases, between which are numerous roots; forming a thick mass; near the camp Selemboekoe, above 180 M. above sea-level: N⁰. 615, 26 VII, 1925.

Philippines, Singkep.

Asplenium Nidus Linnaeus, Spec. plant, II, p. 1079, 1753.

In primary forest in the shadow, scattered, on trees; in the axils of the radially arranged leaves with a mass of roots, between which humus is formed; near camp Selemboekoe, about 180 M. above sea-level: N⁰. 640, 27 VII, 1925; near doesoen Baroe (S. Merangin), about 200 M. above sea-level: N⁰. 1084, IX, 1925.

Tropical Africa, Asia and Australia.

Asplenium tenerum Forster Prodromus, p. 80, 1786.

In the primary forest, on trees, in shadowy spots; B. Mangkok, near the road to S. Manau, about 200 M. above sea-level: N⁰. 669, 5 VII, 1925.

Trop. Africa, Asia, Polynesia.

Blechnum Finlaysonianum Wallich; Hooker et Greville, Icones Filicum pl. 275, 1831.

In native rubber plantations, scattered in rather light spots; near Bangko, about 60 M. above sea-level: N⁰. 468, 9 VII, 1925.

Sumatra, Malacca, Borneo, Java, New Guinea.

Blechnum orientale Linnaeus, Spec. plant., ed. II, p. 1530, 1753.

Between bushes, on left ladangs (dry rice-fields), in newly cleared forest, in native rubber plantations (about 7 years old), scattered; near Bangko, about 60 M. above sea-level: N⁰. 467, 9 VII, 1925; N⁰. 522. 15 VII, 1925.

Tropical Asia, Australia and Polynesia.

Stenochlaena aculeata (Blume) Kunze, Bot. Zeitung, p. 142, 1848.

In the primary forest, in rather humid places; rhizome vertically climbing, on trees; leaves horizontal, adpressed; these specimens belong to the form inermis, the rhizome bearing no spines; the rhizome of N⁰. 902 is thicker than that of the other specimens; S. Merangin, opposite to M. Karing, about 150 M. above sea-level: N⁰. 681, 9 VIII, 1925; near the S. Karing, about 180 M. above sea-level: N⁰. 743, 19 VIII, 1925; S. Selemboekoe, about 150 M. above sea-level: N⁰. 898, 6 IX, 1925; near the camp, about 180 M. above sea-level: N⁰. 902, 903, 26 IX, 1925.

Tropical Asia.

Stenochlaena palustris (Burm.) Beddome, Ferns of Br. India, Suppl. p. 26, 1876.

In secundary jungle, along roads or in rather open places, climbing between shrubs, sometimes rather frequent; near the road from the camp Selemboekoe to Bangko, about 100 M. above sea-level: N⁰. 729, 18 VIII, 1925; near doesoen Baroe (S. Merangin) about 200 M. above sea-level: N⁰. 861, IX, 1925.

Tropical Asia, Australia, Polynesia.

Syngramma valleculata (Baker) C. Christensen, Index Filicum p. 340, 1905.

In primary forest, in groups, on the soil or on the base of old trees; near camp Selemboekoe, about 180 M. above sea-level: N⁰. 617, 26 VI, 1925.

Borneo.

Ceropteris calomelanos (Linnaeus) Underwood, Bulletin Torrey Bot. Club, XXIX, p. 632, 1902.

In open places, along the roads, on clearings in the forest, on sandbanks in the rivers; near Bangko, about 60 M. above sea-level: N⁰. 514, 14 VII, 1925.

All tropical countries.

Cheilanthes tenuifolia (Burmeister) Swartz, Synopsis, p. 129, 332, 1806.

In scattered groups between alang-alang (Imperata cylindrica Beauv.) and along the roads; between Bangko and the camp Selemboekoe, about 100 M. above sea-level: N⁰. 719, 18 VII, 1925.

Tropical Asia, Australia, Polynesia, New Zealand.

Pteridium aquilinum (Linnaeus) Kuhn; von Decken, Reisen, III, Bot. p. 11, 1879.

Along the roads and on clearings in the forest; near Bangko, about 60 M. above sea-level: N⁰. 516, 14 VII, 1925.

All tropical and temperate countries.

Vittaria pumila Mettenius; Kuhn Linnaeus, XXXVI, p. 65, 1869.

In primary forest, epiphytic; near Bangko, about 60 M. above sea-level: N⁰. 588*a*, 20 VII, 1925.

Borneo.

Vittaria scolopendrina (Bory) Thwaites, Enum. Plant. Zeyl. p. 381, 1864.

In primary forest, on trees, in scattered groups; S. Selemboekoe, near the camp, about 180 M. above sea-level: N⁰. 767, 25 VIII, 1925.

Tropical Africa and Australia, Polynesia.

Vittaria zosterifolia Willdenow, Species Plantarum, V, p. 406, 1810.

On the nodes of bamboos, growing near the S. Mesoemai, near Bangko, about 60 M. above sea-level: N⁰. 532, 15 VIII, 1925.

Madagascar, tropical Asia, Polynesia.

Taenites blechnoides (Willdenow) Swartz, Synopsis, p. 24, 220, 1806.

In native rubber-plantations, scattered; near Bangko, about 60 M. above sea-level: N⁰. 453, 9 VII, 1925.

Tropical Asia, Polynesia.

Hymenolepis spicata (L. Fils) Presl. Epim. Bot., p. 159, 1849.

In primary forest; epiphytic. near M. Ketidoeran Siamang, about 140 M. above sea-level: N⁰. 759, 25 VIII, 1925.

Madagascar, tropical Asia, Australia and Polynesia.

Polypodium heterocarpum (Blume) Mettenius. Fil. Hort. Lips. p. 37, pl. 25, fig. 24—25, 1856.

In primary forest, on trees; near M. Ketidoeran Siamang, about 140 M. above sea-level: N⁰. 760, 25 VIII, 1925.

Sumatra. Malacca. Banca. Riouw-Lingga islands. Java, Borneo. Celebes.

Polypodium inconspicuum Blume. Enumeratio. p. 130. 1828.

In primary forest. epiphytic. on humid shadowy places; forming tufts (together with Hymenophyllum subrotundum v. A. v. R.); S. Mengkarang. about 200 M. above sea-level: N⁰. 1082. IX. 1925.

Malay Archipelago, Philippines.

Polypodium nigrescens Blume. Enumeratio, p. 126. 1828.

In primary forest. both in rather open and shadowy places; epiphytic, near S. Mesoemai. near Bangko. about 60 M. above sea-level: N⁰. 534, 15 VII. 1925; S. Merangin. opposite to M. Djangkang, about 140 M. above sea-level; N⁰. 630, 27 VII, 1925.

Tropical Asia and Australia, Polynesia.

Polypodium phymatodes Linn., Mantissa, p. 306, 1771.

In primary forest, epiphytic; near doesoen Baroe. about 200 M. above sea-level: N⁰. 888. IX. 1925.

Tropical Africa, Asia, Australia, Polynesia.

Polypodium revolutum (J. Smith) C. Christensen. Index Filicum. p. 559, 1906.

Epiphytic. in groups on the nodes of bamboos: bamboo bushes on a sandbank. S. Merangin. near M. Titi Meranti; N⁰. 679, 9 VIII, 1925.

Sumatra, Java, Celebes, Philippines, N. Caledonia.

Polypodium Whitfordi Copeland. Phil. Journ. of Science, I. Suppl. V. p. 256, pl. 4 B, 1916.

In primary forest. on trees: S. Ketidoeran Siamang. about 140 M. above sea-level: N⁰. 762, 24 VIII, 1925.

Philippines (Luzon).

Loxogramma Blumeanum Presl. Tentamen Pterid., p. 215, 1836.

In primary forest. on trees: near M. Karing. about 140 M. above sea-level: N⁰. 906. 13 IX. 1925.

Tropical Africa and Asia.

Loxogramma involutum Presl, Tentamen Pterid., p. 215, 1836. (Polypodium scolopendrinum (Bory) C. Christensen, Index Filicum, p. 562, 1906.)

In primary forest, epiphytic, in rather shadowy places; near doesoen Baroe (S. Merangin), about 200 M. above sea-level: N⁰. 884, IX 1925; N⁰. 1083, IX, 1925.

Tropical and subtropical Asia.

Cyclophorus acrostichoides (Desvaux) Presl. Epim. bot. p. 130, 1849.

On the nodes of the stems of bamboos, at the S. Mesoemai, near Bangko, about 60 M. above sea-level: N⁰. 533, 15 VII, 1925.

Ceylon, Malay Archipelago, Queensland, Polynesia.

Cyclophorus angustatus (Swartz) Desvaux, Berliner Magazin, V, p. 300, 1811.

In primary forest, climbing, with a long rhizome on the stems; near Bangko, about 60 M. above sea-level: N⁰. 588, 20 VII, 1925; near S. Karing, about 140 M. above sea-level: N⁰. 610, 25 VII, 1925; S. Selemboekoe, near the camp, about 180 M. above sea-level: N⁰. 781, 25 VIII, 1925.

British India, Sumatra, Malacca, Riouw-Lingga islands, Banca, Borneo, Polynesia.

Elaphoglossum Beccarianum (Baker) C. Christensen, Index Filicum, p. 303, 1925.

In primary forest, on trees, in shadowy places; near M. Ketidoeran Siamang, about 130 M. above sea-level: N⁰. 909, 24 X, 1925.

Borneo.

PARKERIACEAE.

Ceratopteris thalictroides (Linn.) Brongniart, Bull. Soc. Phil., Paris, p. 186, 1821.

Water-fern; Batang Soengai: N⁰. 934, IX, 1925.

All tropical countries.

GLEICHENIACEAE.

Gleichenia linearis (Burm.) Clarke, Trans. Linn. Soc., II, Bot. I, p. 428, 1880.

Along the roads, between shrubs, on clearing in the forest; near Bangko, about 60 M. above sea-level: N⁰. 517, 14 VII, 1925; near camp S. Selemboekoe, about 180 M. above sea-level: N⁰. 765, 25 VIII, 1925.

Trop. countries.

SCHIZAEACEAE.

Lygodium circinnatum (Burm.) Swartz., Synopsis Filicum, p. 153, 1806.

Along the roads and between shrubs, near the camp Selemboekoe, about 160 M. above sea-level: N⁰. 727, 18 VIII, 1925; near M. Karing, about 140 M. above sea-level: N⁰. 603, 25 VII, 1925; near doesoen Baroe (S. Merangin) about 200 M. above sea-level: N⁰. 860, IX, 1925.

Tropical Asia, Queensland.

Lygodium flexuosum (Linn.) Swartz, Schraders Journ., p. 106, 1801.

Between alang-alang (Imperata cylindrica Beauv.) on open places; along the road to Bangko, about 180 M. above sea-level: N⁰. 717, 718, 18 VIII, 1925.

Tropical Asia, Queensland.

Lygodium salicifolium Presl, Suppl. Tent. Pterid., p. 102, 1845.

In native rubber-plantations, near Bangko, about 60 M. above sea-level: N⁰. 464, 9 VII 1925.

Tropical Asia.

MARATTIACEAE.

Angiopteris evecta Hoffmann, Commentatio Soc. Reg. Gött., XII, p. 29, pl. 5, 1796.

Between shrubs (secundary jungle) near the river-banks; S. Tembesi near Sarolangoen, about 40 M. above sea-level: without number, 1 VII, 1925.

Tropical Asia.

OPHIOGLOSSACEAE.

Helminthostachys zeylanica (Linn.) Hooker, Gen. Filicum, pl. 47, 1840.

Near doesoen Baroe (S. Merangin), about 200 M., above sea-level: N⁰. 867, IX, 1925.

Tropical Asia and Australia.

LYCOPODIACEAE.

Lycopodium cernuum Linnaeus, Species plantarum, I, p. 1103, 1753.

In native rubber-plantations, about 7 years old, without undergrowth, scattered in rather shadowy places; near Bangko, about 60 M. above sea-level: N⁰. 466, 7 VII, 1925.

Tropical and subtropical countries.

Lycopodium Phlegmaria Linnaeus, Species plantarum, I, p. 1101, 1753.

At the margin of the primary forest, epiphytic, pendulous; near the road to S. Manau near the camp Selemboekoe; about 180 M. above sea-level: N⁰. 796, 26 VIII, 1925.

Tropical Asia and Australia.

Lycopodium pinifolium Blume, Enumeratio, p. 264, 1828.

In primary forest, epiphytic, pendulous ; near M. Ketidoeran Siamang, 140 M. above sea-level : N⁰. 910, 24 X, 1925.

Malay islands, New Guinea.

PSILOTACEAE.

Psilotum complanatum Swartz, Synopsis, p. 414, pl. 4, fig. 5, 1806.

In primary forest, hanging down from below a nest-fern, Asplenium Nidus ; S. Karing, about 140 M. above sea-level : N⁰. 821, 28 VIII, 1925.

Tropical and subtropical countries.

SELAGINELLACEAE.

Selaginella alopecuroides Baker, Handbook Fern Allies, p. 77, 1887.

On shadowy places, near the water side ; S. Lesing, near Paoe, about 30 M. above sea-level : N⁰. 1006, X, 1925.

Borneo.

Selaginella atroviridis Spring, Monographie, II, p. 124, 1848.

In primary forest, on the soil or on the rocky slopes of small rivers, in shadowy places ; M. Karing, about 140 M. above sea-level : N⁰. 644, 1 VIII, 1925 ; path to Ds. Baroe (S. Merangin), about 180 M. above sea-level : N⁰. 666, 4 VIII, 1925.

Tropical Asia.

Selaginella convolvens van Alderwerelt van Rosenburgh, Bulletin Jard. Bot., Buitenzorg XI, p. 23, 1913.

In primary forest, op humid rocky slopes, in shadow ; M. Karing, about 140 M. above sea-level : N⁰. 605, 25 VII, 1925.

Tropical Asia.

Selaginella involvens Hieronymus, Hedvigia, L. p. 2, 1911.

In primary forest, op humid rocky slopes, in shadow ; M. Karing, about 140 M. above sea-level : N⁰. 609, 25 VII, 1925.

Tropical Asia.

Selaginella phanotricha Baker, Handbook Fern Allies, p. 109, 1887.

In primary forest on the soil or on the base of old trees in shadow ; S. Selemboekoe, about 180 M. above sea-level : N⁰. 772, 25 VIII, 1925.

Borneo.

Selaginella plana Hieronymus in Engler-Prantl, Nat. Pflanzenfam. I. abt. IV, p. 704, 1900.

In primary forest, on rather open, rocky places ; also in native rubber-plantations ; near Bangko, about 60 M. above sea-level : N⁰. 458, 9 VII, 1925 ; S. Merangin, opposite to M. Djangkong, about 140 M. above sea-level : N⁰. 619, 27 VII, 1925.

Tropical Asia.

Selaginella Willdenowii Baker, Handbook Fern Allies, p. 93, 1887.

On shadowy places, near the water side; S. Lesing, near Paoe, about 30 M. above sea-level: N⁰. 1009, X, 1925.

Tropical Asia.

The collection which is described on the preceding pages, represents the fern-flora of a relative small district only, in the hilly part east of the Barisan Mountains. Though therefore the conclusions derived from its study have not general value for the fern flore of Sumatra as a whole, still some remarks may be made here.

The analysis of the vegetation of a rather small district, in which the conditions are not differing very much, has, to a certain degree, some advantages to the study of the flora of a country, like Sumatra, as a whole. The circumstances governing the distribution of plants, find also their expression in the relation of the flora of a small district with that of other countries.

From the 74 species of Ferns, the majority (two thirds) is found throughout the Malay Archipelago in the plains and the lower regions of the mountains: not only on Sumatra, Malacca, Borneo, Celebes and the Moluccas, but also on Java; many of them are also known from British India, China and N. Australia, some of them occur in Madagascar and Polynesia, a number even in nearly all tropical countries.

From the remaining 22 species the known distribution is the following:

Sumatra, Malacca, Borneo, Celebes and the Moluccas:

Cibotium Barometz (Linn.) J. Smith (also known from Formosa, Hongkong, China).

Cyathea moluccana R. Brown (also known from Assam).

Dryopteris didymosora (Parish) C. Christ. (new for Sumatra; also known from Assam Br. India and S. China).

Aspidium angulatum (Willd.) J. Smith (known from New Guinea).

Cyclophorus angustatus (Swartz) Desvaux (also known from Br. India and Polynesia).

Cibotium barometz has not been mentioned from Java in the literature, neither were specimens from not-cultivated plants from this island present in the Buitenzorg herbarium; it has been collected, however by TEYSMANN on the mount Radjabasa in the S. E. part of Sumatra, on the point nearest to Java (specimen in the Buitenzorg herbarium). *Cyclophorus angustatus* has been mentioned with doubt from Java by RACIBORSKY [1]); this statement has never been confirmed.

Sumatra and the Philippines:

Asplenium cymbifolium Christ. (new to Sumatra, already known from Singkep) [2]).

Polypodium Whitfordii Copeland (new to Sumatra).

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[1]) RACIBORSKI, Pteridophyta von Buitenzorg, 1898, p. 100.

[2]) VAN ALDERWERELT VAN ROSENBURGH, Bull. Jard. Bot. de Buitenzorg (3) V, 1922, p. 184.

Sumatra, Malacca, Borneo :
Trichomanes hispidulum Mettenius (new to Sumatra).
Dryopteris salicifolia (Wall.) C. Chr.
Aspidium singaporianum Wallich.
Humata angustata J. Smith.

Sumatra, and Borneo (N.-W. and N. Borneo) :
Dryopteris sarawakensis (Baker) v. A. v. R. (new to Sumatra).
Diplazium aequibasale (Baker) C. Chr. (new to Sumatra).
Diplazium confertum (Baker) C. Chr. (also known from Celebes) [1]).
Syngramma valleculata (Baker) C. Chr. (new to Sumatra).
Vittaria pumila Mett. (new to Sumatra).
Elaphoglossum Beccarianum (Baker) C. Christ. (new to Sumatra).

Sumatra and Malacca :
Trichomanes singaporianum (v. d. Bosch) v. A. v. R. (new to Sumatra).
Dryopteris Dayi (Beddome) C. Chr. (new to Sumatra).
Davallia triphylla Hooker (new to Sumatra).

From Sumatra only are known :
Hymenophyllum subrotundum v. A. v. R.
Aspidium nebulosum (Baker) C. Chr. (also from Banca).

Thus we see a rather high number of species which are found rather far eastward, but appear to be absent on Java ; on the contrary only two species of the collection are known from Java and Sumatra, only :
Trichomanes sumatranum v. A. v. R.
Dryopteris verruculosa v. A. v. R.

These species are both rare on Java ; *Trichomanes sumatranum* has been found once near Buitenzorg [2]); *Dryopteris verruculosa* once near Srigontjo at the south coast, S. of Bantoer (Malang) [3]); the specimens of the later species differ slightly from those gathered in Bencoolen (Lebong Tandai) and in Djambi.

From the Ferns, 20 species occurred in secundary vegetation ; except *Cyathea moluccana* and *Dryopteris didymosora* they all belonged to the species, which Sumatra has in common with Java, and which are found also throughout the Malay Archipelago, sometimes even far without ; even *Cyathea moluccana* and *Dryopteris didymosora*, which have not been found on Java, have a rather large distribution, from British India and Assam to the Moluccas.

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[1]) C. CHRISTENSEN, Svensk. bot. Tidskr., XVI, 1922, p. 93.

[2]) VAN ALDERWERELT VAN ROSENBURGH, Bull. Jardin Bot. de Buitenzorg (3) V, 1922, p. 226.

[3]) VAN ALDERWERELT VAN ROSENBURGH, Bull. Jard. Bot. Buitenzorg, XI, 1913, p. 12; id., (3) II, 1920, p. 150.

In addition to the above remarks concerning the secundary vegetation, the following can be said concerning the species, which are found nearly throughout the whole Archipelago, also on Java.

When comparing the flora of Java and Sumatra, one is struck by the fact, already mentioned by MIQUEL [1]), that a number of plants, which in Java are known at rather high altitudes only, occur in Sumatra also in the plains. This is already stated for the Ferns bij RACIBORSKY [2]), who remarks that the lowest localities of many species are found on Sumatra at a lower level than on Western Java; on Mid Java (Tegal) they are found at a still higher altitude.

This is also demonstrated for some of the ferns mentioned above; the bulk of the species which Sumatra has in common with Java occurs there in the plains too; but a number of them are known in Java only from localities situated at a higher altitude than those on Sumatra; from these forms the following list is given; the numbers in brackets placed behind the names indicate the altitude of the lowest localities on Java of the species, which are known to me.

Hymenophyllum holochilum (v. d. Bosch) C. Christ. (900 M.).
Trichomanes sumatranum v. A. v. R. (750 M.).
Dipteris conjugata Reinwardt (900 M.).
Dryopteris calcarata (Blume) O. K. (500 M.).
Didymochlaena truncatula (Swartz) J. Sm. (700 M.).
Tapeinidium gracile v. A. v. R. (900 M.).
Lindsaya decomposita Willd. (800 M.).
Lindsaya lancea (Linn.) Bedd. (600 M.).
Lindsaya scandens Hooker (800 M.).
Diplazium bantamense Blume (1000 M.).
Asplenium tenerum Forst. (500 M.).
Blechnum Finlaysonianum Wallich (1000 M.).
Polypodium heterocarpum (Blume) Mett. (900 M.).
Polypodium inconspicuum Blume (1650 M.).
Polypodium revolutum (J. Sm.) C. Christ. (450 M.).

This difference is probably chiefly due to the fact, that the rainfall is here more equally distributed throughout the year than on Java [3]). There generally the plains have a more pronounced dry season than most of the mountainous parts. If the above supposition be true, we might expect these forms to occur on Java also in those parts of the plains or in hilly districts where the rain is equally distributed in the way: in the remote western parts of the Preanger Residency (near the Wynkoopsbaai), in the Eastern part of the Preanger, near the South coast (W. of Tjidoelang), and on the southern slope of the Smeroe. The flora of these regions, however, is hardly known.

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[1]) MIQUEL, Flora van Sumatra, 1860, p. 38.

[2]) RACIBORSKI, Farne von Tegal, 1899, p. 235.

[3]) The importance of this fact, more especially for the culture of tea on Java, has been demonstrated by BACKER and VAN SLOOTEN, Theeonkruiden, 1924, p. 13; map on p. 12.

From the above mentioned species only two, *Blechnum Finlaysonianum* and *Polypodium revolutum* occur, in Djambi, in the secundary vegetation. They belong, however, to the group, which contains nearly all ferns which may occur in the secundary vegetation. This phenomen can be explained by the supposition that the lesser resistance against drought is here the limiting factor. Then the species can not live on Java in the temporary rather dry plains in the secundary vegetation, where usually rather much resistance is necessary, especially in the young plants. The extremes of these conditions are for the plants more important than the total amount of the rainfall during longer periods.

The fern-flora of Java is rather well known, relatively much better than that of Borneo and Sumatra. The bulk of the species, mentioned above, are found throughout the whole Archipelago and even far without. We see that from the remaining species Sumatra has only some rare ones in common with Java; the other ones are not found on Java, but, however, have a rather large extension in eastward direction.

The same phenomenon is even valid for genera e.g. for *Dipteris, Matonia, Syngramma, Lecanopteris* and some families of higher plants. The species, which Java and Sumatra have in common, are found throughout the Archipelago; but those species with a more restricted distribution are found in Sumatra and Malacca, Borneo only.

The conclusions got by the analysis of this collection of ferns gathered in a rather small district in the hilly part of the lowlands of Mid Sumatra, are in accord with those derived from the study of the distribution both of several groups of higher plants and of the fauna. Though Sumatra and Java are separated only by the rather narrow straits of Sunda, their flora and fauna appear to be much more different than the flora and fauna of Sumatra and Malacca, even more different than the flora and fauna of Sumatra and Borneo.

For the discussion of this fascinating subject the reader may be referred to the paper of Dr. H. J. LAM [1]), who has checked the conclusions, derived from the distribution of the fauna with those of some groups of plants especially the Sapotaceae. The above conclusions are in accord with his. Also there the Sunda straits appears to be a boundary of primary importance; even more than the "line of Wallace". The line through the straits of Sunda is identical with the western part of the line of "Van Kampen", that is the eastern boundary of the area of many animals, among which most large mammals [2]). The supposition that this boundary really is the straits of Sunda and not, for instance, is situated some hundred miles towards the north, is supported by the occurrence of some plants in the southern Lampongs which have not been found in Java. From the literature may be cited here: *Anisoptera marginata* Korthals (from Sumatra, Malacca and Borneo) found near Tandjoeng Karang, near Telok Betoeng [1]; from the Ferns the occurrence of *Cibotium barometz* J. Sm. on the Radjabasa, mentioned above. Further collecting in these districts will doubtless give valuable information concerning these questions.

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[1]) Annales Jard. Bot. Buitenzorg, XXXVII, p. 33—49, 16 pl.

[2]) l.c., p. 42.

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[1] VAN SLOOTEN, Bull. Jardin Bot. de Buitenzorg (3) VIII, 1926, p. 7.

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O. POSTHUMUS, Eenige opmerkingen betreffende de palaeozoische Flora van Djambi, Sumatra. Zittingsverslagen Kon. Akad. van Wetenschappen. Amsterdam, XXXVI, 1927, p. 428—434 and Proceedings Royal Acad. Amsterdam, **30**, 1927, p. 628.

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Chemistry. — *Osmosis of ternary liquids.* Experimental part I. By Prof. F. A. H. SCHREINEMAKERS and Mr. B. C. VAN BALEN WALTER.

(Communicated at the meeting of January 28, 1928).

In previous communications, to be continued later on, one of us[1]) has discussed the osmosis of two ternary liquids in osmotic contact with one another with the aid of a membrane permeable for each of the three substances. We are now going to discuss a group of the systems examined.

We take the liquids L_1 and L'_1 of which the first contains only $Na_2CO_3 + H_2O$ and the second $NaCl + H_2O$ only. In fig. 1 we draw the $NaCl$-amount of these liquids on the horizontal axis and on the vertical axis their Na_2CO_3 amount; then L_1 is represented by a point 1 on the vertical axis and L'_1 by a point $1'$ on the horizontal axis.

If we bring both liquids in osmotic contact with one another, then we have at the moment $t = 0$ viz. at the beginning of the osmosis, the system:

$$L_1 \mid L'_1 \qquad \ldots \ldots \ldots \ldots (1)$$

This travels along an osmosis-path, as has been discussed before; in order to determine the form of this path, we at intervals took away a little of both liquids to analyse them; consequently we did not determine the theoretical path, but an experimental path[2]) of the system.

As may be seen in the tables I—V, the membrane in the systems I—IV, consisted of a pig's bladder; this had first been degreased with aether; the membrane of system V was of parchment.

The paths of the systems I and V have been represented schematically in fig. 1 by the curves I and V; with the aid of the tables they can be accurately drawn. As the starting-points 1 of both paths coincide approximately on the Y-axis, they have in fig. 1 been drawn as coinciding.

Each path consists of the two conjugated branches[3]) $1 \,.\, e$ and $1' \,.\, e$. Consequently the left side liquid of system (1) travels along the branch $1 \,.\, e$ and the right side liquid travels along the branch $1' \,.\, e$ in a direction, which has been indicated by the arrows.

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[1]) F. A. H. SCHREINEMAKERS. Osmosis of ternary liquids. General considerations I, II, III and IV. These Proceedings **30**, 761. In future they will be quoted as: Gen. I, Gen. II, etc.

[2]) Gen. I.

[3]) Gen. I.

The paths of the systems II, III and IV have not been drawn in fig. 1; approximately the points 1 of those paths coincide with point 1 in fig. 1; the points 1′ of paths II and III are situated between those of I and V; the point 1′ of path IV coincides approximately with that of V.

Although, therefore, the paths IV and V approximately have the same starting-points 1 and 1′, yet their paths do not coincide; we shall refer to this later on.

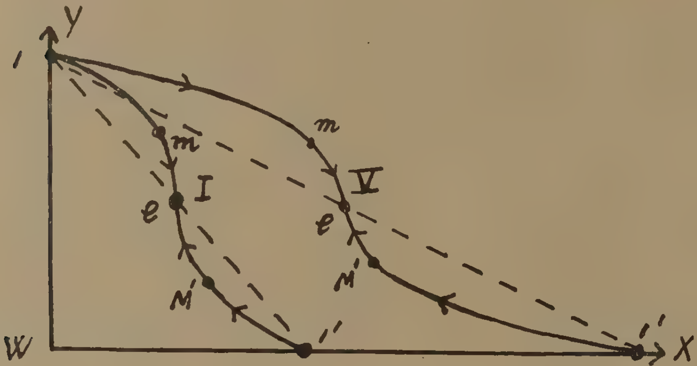

Fig. 1.

In the first column of the tables I—V we find the number of the successive determinations; sub *t* we find the time, viz. the number of hours after the beginning of the osmosis. In the third and fourth columns we find sub *X* the *NaCl*-amount and sub *Y* the Na_2CO_3-amount of the left side liquid; in the fifth and sixth columns we find the same for the right side liquid. The concentrations have been indicated in procents of weight. As the amount of water follows at once from the *X*- and *Y*-amounts, this has been indicated only for the systems I and II in the tables I^a and II^a. We shall refer later on to the meaning of the other columns.

In the schemes I—V the apparent and the real osmosis[1]) of the systems have been represented. The real osmosis, that is to say the direction in which the substances diffuse through the membrane, has been represented by the horizontal arrows; the apparent osmosis viz. the changes in concentration of each of the three substances, has been indicated by the vertical arrows. As we shall first discuss the apparent osmosis, the reader had better neglect the horizontal arrows to begin with.

It appears from the form of path I in fig. 1 and from table I that the X-amount (so $NaCl$) of the left side liquid is smaller than that of the right side liquid during all the osmosis; besides we see that the X-amount of the left side liquid continuously increases and that the X-amount of the right side liquid continuously decreases. This has been represented in scheme I sub X by the symbol:

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[1]) Gen. II and III.

$\uparrow < \downarrow$ . . . . (2)

As this is also the case with the other systems, as is apparent from the tables, we may say, therefore:

the X-amount of those systems changes during all the osmosis normally-normally.

We can also represent the change of the X-amount in an $X.t$-diagram (Gen. II); in order to do this we draw the time t of the osmosis on the horizontal axis and the X-amount of both liquids on the vertical axis.

If with the aid of table I we draw this diagram for system I, we get a figure of the same type as fig. 2 (Gen. II); in this the $X.t$-path of the left side liquid has been represented by the curve 1 . e, which starts from point O and that of the right side liquid by curve 1′. e. In order to facilitate a general view of the subject (just as in the corresponding other cases) the path of the left side liquid has here been fully drawn and that of the right side liquid has been dotted. Both curves go towards the point e, situated at infinite distance, which indicates the W-amount of the final liquid e.

From the course of these curves it appears also that the X-amount changes normally-normally on both sides of the membrane; in the figure this has been indicated by the letters N.

It is easy to see that also the $X.t$-diagrams of the other systems may be represented schematically by fig. 2 (Gen. II).

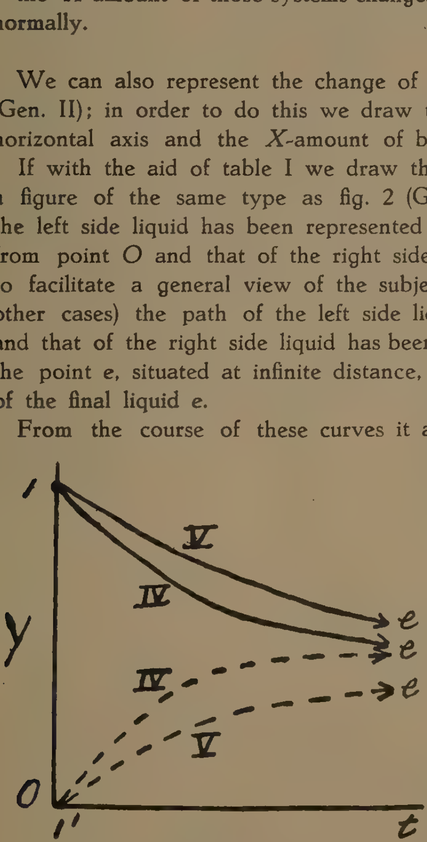

Fig. 2.

We now consider the Y-amount (Na_2CO_3) of system I. It appears from fig. 1 and table I that during all the osmosis this Y-amount is larger on the left side of the membrane than on the right side; we also see that the Y-amount on the left side decreases continuously and that it increases on the right side. This has been represented in scheme I sub Y by the symbol:

$\downarrow > \uparrow$ . . . . (3)

As this is also the case for the other systems, we may say therefore:

the Y-amount of these systems changes: normally-normally during all the osmosis.

In fig. 2 the Yt-diagram of the systems IV and V has been represented schematically; the correct form of these paths may be easily drawn with the aid of the tables IV and V. The Yt-paths of the other systems have a corresponding form, viz. ascending when starting form point 1′ and descending from point 1. As is apparent from the tables the beginning-points 1 of the paths coincide approximately.

The symbol (3) also follows at once from this diagram. Formerly we have already discussed the Yt-diagram of fig. 3 (Gen. II); in this the path of the left side liquid has a maximum, which does not occur in fig. 2.

We now consider the W-amount of system I; for this purpose we use table I^a in which the W-amount of this system has been given. We see that the W-amount of the left side liquid begins by decreasing till the determination noted with a † and increases afterwards. Consequently a minimum W-amount is situated in the vicinity of 3, which we shall call m; this is situated between 2 and 3 or between 3 and 4; therefore, certainly between 2 and 4.

The W-amount of the right side liquid increases till the determination noted with † and decreases afterwards. Consequently the maximum W-amount, which we shall call M' is situated somewhere between 2 and 4 (2′ and 4′).

For clearness' sake we begin by imagining that m and M' coincide with determination 3. We then see that on the left side of the membrane from 1 to 3 the W-amount is smaller than on the right side and that it yet continues to decrease on the left and increase on the right. So for part 1.3 (and 1′.3′) of the path the symbol:

$$^{*}\!\downarrow\, < \,\uparrow^{*} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

obtains, which we also find sub W in scheme I. During the osmosis from 1 to 3 the W-amount of system I changes, therefore, anormally-anormally.

It appears from the determinations 3 to 7 that the W-amount increases on the left side of the membrane and that it decreases on the right side; as the W-amount is smaller on the left side than on the right side, the symbol:

$$\uparrow\, < \,\downarrow \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

will follow, which we also find in scheme I for the parts 3.5 and 5.7, consequently for part 3.7.

From (4) and (5) follows: during the osmosis the W-amount of system I changes on the part:

1—3 of its path: anormally—anormally
3—7 „ „ „ : normally—normally.

The above has been deduced, assuming that the points m and M' coincide with 3; as this is generally not the case of course, there are transitions between the symbols (4) and (5). Although we are able to deduce them at once from tab. I^a, we shall do this with the aid of the $W.t$-diagram.

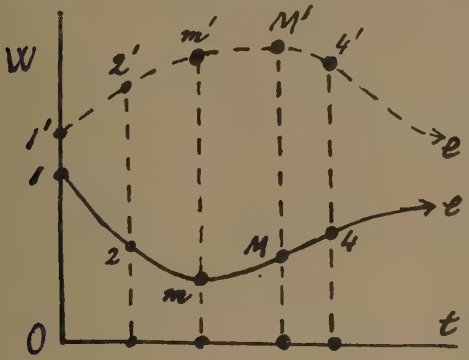

Fig. 3.

We see from table I^a that the path of the left side liquid can be represented schematically by curve $1.2.4.e$ and that of the right side liquid by curve $1'.2'.4'.e$ of fig. 3 [1]; the minimum, point m is situated somewhere between 2 and 4; the maximum, point M' between $2'$ and $4'$. In this figure we have assumed that during the osmosis first the left side liquid reaches its minimum m and afterwards the right side liquid its maximum M'. We see from this:

on $1 . m$ $(1' . m')$ is valid: the symbol $^*\!\downarrow\, < \,\uparrow\!^*$
on $M e$ $(M' . e)$ „ „ „ „ $\uparrow\, < \,\downarrow$.

In point m the W-amount of the left side liquid is a minimum; so it remains constant during an infinitely small time dt; in point m', however, it increases in that time dt; we represent this by:

$$|\, < \,\uparrow\!^* .$$

As the W-amount on the left side of the membrane does not change, the left arrow has been substituted by a dash.

On part mM of the path the W-amount of the liquid increases; on part $m'M'$ the W-amount also increases; consequently we find the symbol:

$$\uparrow\, < \,\uparrow\!^*$$

so that the W-amount now changes normally-anormally.

For the point $M\,(M')$ we find:

$$\uparrow\, < \,|$$

Consequently we get the successive symbols:

$$^*\!\downarrow\, < \,\uparrow\!^* , \quad |\, < \,\uparrow\!^* , \quad \uparrow\, < \,\uparrow\!^* , \quad \uparrow\, < \,| \,; \quad \uparrow\, < \,\downarrow \quad . \; . \; . \quad (6)$$

from which we see how the first passes into the last.

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[1] In drawing this figure we may imagine all concentrations given in tab. II^a reduced by a definite number e.g. 90; naturally the course and mutual position of the paths are not changed in this way.

If we assume that during the osmosis the maximum M' is found first on the right side of the membrane and afterwards the minimum m on the left side of the membrane, then we find:

$${}^{*}\!\downarrow < \uparrow^{*}, \quad {}^{*}\!\downarrow < |, \quad {}^{*}\!\downarrow < \downarrow, \quad | < \downarrow, \quad \uparrow < \downarrow \quad . \; . \; . \quad (7)$$

Consequently we find: the W-amount of system I changes:

a. at the beginning of the osmosis: anormally-anormally.

b. afterwards: normally-normally.

During the transition from *a* to *b* it changes according to (6), so normally-anormally, or according to (7), so anormally-normally.

In the schemes I—V only the symbols resulting from the determinations have been given. The reader will have no trouble in seeing where transitions will occur and how they are to be found; we shall refer later on to some special cases.

In table II^a we find the W-amount of the liquids of system II; it appears from this that during the osmosis from 1 to 5 the W-amount on the left side is larger than that on the right side and that it decreases on the left and increases on the right.

Consequently for this part 1.5 obtains the symbol:

$$\downarrow > \uparrow \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

Therefore, the W-amount changes normally-normally. Although this symbol is valid for part 1.5 of the path and not for all points between 5 and 6, yet it has been indicated in scheme II for the whole of part 1.6; presently we shall see that between 5 and 6 a transition occurs to the next symbol.

From 6 to 9 the W-amount on the left side of the membrane is smaller than that on the right side. If we limit again ourselves to the values found, we see that in the determinations noted with a † the W-amount is a minimum on the left side and a maximum on the right side. Consequently for part 6.8 of the path obtains the symbol:

$${}^{*}\!\downarrow < \uparrow^{*} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

and for part 8.9:

$$\uparrow < \downarrow \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

which pass into one another either according to (6) or according to (7).

We now consider the transition between the symbols (8) and (9). It appears from table II^a that between 5 and 6 the liquids get the same W-amount; this has been indicated by the horizontal line $q\,q'$; we shall call those points of the path also q and q'. As in a time dt the W-amount of the left side liquid decreases and that of the right side liquid increases, we consequently have the symbol:

$${}^{*}\!\downarrow = \uparrow^{*} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

So we have two liquids with the same *W*-amount, which, however, changes again during the osmosis, viz. first according to (9) and (10) again to become the same in point *e*.

From the preceeding discussions it follows: the *W*-amount of system II changes:

a. in the beginning of the osmosis: normally-normally.
b. afterwards: anormally-anormally.
c. at last again: normally-normally.

Between *a* and *b* is situated a transition according to symbol (11) and between *b* and *c* according to (6) or (7).

The above may also be easily deduced from the *W.t*-diagram; if we draw this with the aid of table II^a, we see that the path of the left side liquid may be represented schematically by curve 1.2.3.4.*e* and that of the right side liquid by curve 1.′2.′3.′4.′*e* of fig. 4 (Gen. II)[1]. The point of intersection of the curves represents the point *q* and *q*′ where both liquids have the same *W*-amount; in this figure the minimum is situated on the left side of the maximum, but of course it can also be situated on its right.

From tables III—V we are able to deduce the *W*-amount of these systems, their schemes, the transitions occurring in them and their *W.t*-diagrams. These diagrams belong to the type of fig. 4 (Gen. II), although the positions of the point of intersection, the minimum and the maximum may vary greatly with respect to one another.

Comparing fig. 3 with fig. 4 (Gen. II), we see that the latter can pass into the former when the point of intersection drops out. We now find: the *W*-amount of the systems II—V changes normally-normally as well at the beginning as at the end of the osmosis; between them a series of transitions occurs which the reader can deduce for each separate case.

Previously we have seen (Gen. II) that we may also deduce the change of the *W*-amount of a system from its path in fig. 1.

It is possible namely to draw a tangent in each of these paths in a point *m* of branch 1 . *e* and in a point *M*′ of branch 1′. *e*, which cuts equal parts off the *X*- and *Y*-axis and consequently runs parallel to the side *XY*.

So the *W*-amount decreases on branch 1 . *m* . *e* from 1 to *m* and afterwards it increases from *m* to *e*; on branch 1′*M*′*e* it increases from 1′ to *M*′ and afterwards it decreases from *M*′ to *e*. Consequently the *W*-amount is a minimum in *m* and a maximum in *M*′.

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[1] It is clear that the figures 2, 3 etc. here do not represent the corresponding determinations of table IIa.

Through point *e* of each of the paths II—V (of wich only V has been drawn) we may draw a line, which runs parallel to side *XY* and intersects the path in two points; these points represent the conjugated liquids *q* and *q′* with the same *W*-amount.

Of course we can also draw a line through point *e* of path I, running parallel to *XY*; as, however, this line does not intersect the path no points *q* and *q′* will be found here.

With the aid of these data the reader can now deduce the change of the *W*-amount from these paths also.

Tables IV and V show that at the beginning of the osmosis both liquids of system IV have approximately the same composition as those of system V; so the points 1 ond 1′ of both paths coincide approximately in fig. 1. This, however, is not the case with the paths themselves; this is indeed evident, as in IV a pig's bladder and in V a piece of parchment had served as membrane.

We also see from the tables that the liquids in system IV change their composition much more quickly than those of V.

If after an osmosis of 77 hours we compare e.g. the *X*-, *Y*- and *W*-amounts of the left side liquid of system IV with those of system V, we find:

the *X*-amount of IV has increased with: 10.306 %
" " " " V " " " 4.805 %

the *Y*-amount in system:

IV has decreased with 8.552 — 6.666 = 1.886 %
V " " " 8.435 — 7.950 = 0.485 %

the *W*-amount in system:

IV has decreased with 91.448 — 83.028 = 8.420 %
V " " " 91.565 — 87.245 = 4.320 %.

One gets a clearer impression of these differences by drawing the *Xt*-, *Yt*- and *Wt*-diagrams of these systems; for the *Yt*-diagram we then get fig. 2 in which the paths have been drawn schematically. From this we see that the *Y*-amount of the left side liquid (curve 1 . *e*) decreases more quickly in system IV than in V and that the *Y*-amount of the right side liquid (curve 1′. *e*) increases more quickly in system IV than in V.

It is clear that these phenomena will always occur when different membranes are used. As, however, all pig's bladders etc. also differ the one from the other, differences are to be expected here too, and very large they will sometimes be, as we shall discuss later on.

TABLE I. A pig's bladder. Procent of weight.
$X = NaCl$ $Y = Na_2CO_3$ $W = H_2O$

	t	X	Y	X	Y	X	Y	W		
1	0	0	8.654	8.477	0					
2	24	1.195	7.901	7.285	0.7281	58.1	33.9	8.0	VI	←
3	72	2.681	6.724	5.770	1.905	47.8	35.3	16.9	„	„
4	139	3.651	5.680	4.819	2.961	21.6	28.7	49.7	IV	→
5	193	3.958	5.145	4.526	3.483	10.2	27.3	62.5	„	„
6	241	4.045	4.839	4.399	3.780					
7	285	4.143	4.657	4.333	3.969					

TABLE Ia.

	W	W
1	91.346	91.523
2	90.904	91.987
3	+ 90.595	92.325 +
4	90.669	92.210
5	90.897	91.991
6	91.115	91.821
7	91.200	91.698

SCHEME I.

	X	Y	W
1—3	$\underset{\longleftarrow}{\uparrow < \downarrow}$	$\underset{\longrightarrow}{\downarrow > \uparrow}$	$\underset{\longleftarrow}{{}^{*}\downarrow < \uparrow^{*}}$
3—5	„	„	$\underset{\longrightarrow^{*}}{\uparrow < \downarrow}$
0 5—7	$\uparrow < \downarrow$	$\downarrow > \uparrow$	$\uparrow < \downarrow$

TABLE II. A pig's bladder. Procent of weight.
$X = NaCl$ $Y = Na_2CO_3$ $W =$ *water*

	t	X	Y	X	Y	X	Y	W		
1	0	0	8.637	17.584	0					
2	7	0.9376	8.435	16.771	0.2155	30.8	9.9	59.3	IV	→
3	20	2.624	8.006	15.109	0.6123	60.4	13.6	26.0	VII	←
4	44	4.954	7.278	12.843	1.273	68.9	18.2	12.9	„	„
5	68	6.401	6.685	11.428	1.816	66.5	22.1	11.4	VI	„
6	92	7.317	6.160	10.472	2.298	32.1	10.2	57.7	„	„
7	118	7.839	5.738	9.870	2.732	21.8	7.3	70.9	„	„
8	165	8.397	5.198	9.272	3.315	36.5	38.9	24.6	IV	→
9	214	8.601	4.856	9.072	3.728	8.2!	5.5!	86.3!	I	→

TABLE IIa.

	W	
1	91.363	82.416
2	90.627	83.014
3	89.370	84.279
4	87.768	85.884
5	86.914	86.756
6	q 86.523	87.230 q′
7	86.423	87.398
8	+ 86.405	87.413 +
9	86.543	87.200

SCHEME II.

	X	Y	W
1—4	$\underset{\longleftarrow}{\uparrow<\downarrow}$	$\underset{\longrightarrow}{\downarrow>\uparrow}$	$\underset{\longrightarrow}{\downarrow>\uparrow}$
4—6	„	„	$\underset{\longrightarrow *}{\downarrow>\uparrow}$
6—7	„	„	$\underset{\longrightarrow}{*\downarrow<\uparrow*}$
7—8	„	„	$\underset{\longrightarrow *}{*\downarrow<\uparrow*}$
0 8—9	$\underset{\longrightarrow *}{\uparrow<\downarrow *}$	„	$\underset{\longrightarrow *}{\uparrow<\downarrow}$

TABLE III. A pig's bladder. Procent of weight.

$Y = NaCl$ $Y = Na_2CO_3$ $W = H_2O$

	t	X	Y	X	Y	X	Y	W		
1	0	0	8.559	21.026	0					
2	6	0.7965	8.438	20.162	0.1461	42.9	7.7	49.3	IV	→
3	15	2.128	8.228	18.834	0.386	40.3	8.0	51.7	„	„
4	29	3 881	7.877	17.126	0.718	53.4	10.1	36.5	VII	←
5	53	6.322	7.285	14.836	1.271	44.7	10.8	44.5	„	„
6	78	7.893	6.746	13.394	1.728	77.1	21.0	1.9	VI	„
7	120	9.286	5.947	11.883	2.425	26.7	5.3	68.0	„	„
8	169	10.026	5.304	11.287	3.044	4.5	10.1	85.4	I	→
9	217	10.256	4.908	11.023	3.464	12.0	2.0	86.0	„	←

SCHEME III.

	X	Y	W
1—5	$\underset{\longleftarrow}{\uparrow<\downarrow}$	$\underset{\longrightarrow}{\downarrow>\uparrow}$	$\underset{\longrightarrow}{\downarrow>\uparrow}$
5—7	„	„	$\underset{\longleftarrow *}{\downarrow>\uparrow}$
0 7—8	$\underset{\longrightarrow *}{\uparrow<\downarrow}$	„	$\underset{\longrightarrow}{*\downarrow<\downarrow}$
0 8—9	$\underset{\longleftarrow}{\uparrow<\downarrow}$	$\underset{\longleftarrow *}{\downarrow>\uparrow}$	$\underset{\longleftarrow}{\uparrow<\downarrow}$

TABLE IV. A pig's bladder. Procent of weight.
$X = NaCl$ $Y = Na_2CO_3$ $W = H_2O$

	t	X	Y	X	Y	X	Y	W		
1	0	0	8.552	26.773	0					
2	5½	1.029	8.427	25.802	0.166	30.8	7.0	62.2	IV	→
3	13	2.389	8.225	24.405	0.358	56.4	7.3	36.3	VII	←
4	26	4.979	7.878	22.139	0.701	46.3	7.3	46.4	IV	→
5	50	7.958	7.281	19.204	1.233	55.2	9.4	35.3	VII	←
6	77	10.306	6.666	17.038	1.770	63.1	14.1	22.8	,,	,,
7	120	12.183	5.902	15.276	2.466	67.1	23.6	9.3	,,	,,
8	167	12.952	5.302	14.456	3.043	25.2	6.1	68.7	VI	,,
9	241	13.415	4.772	14.062	3.612	12.7	5.6	81.7	I	→

SCHEME IV.

	X	Y	W
1—7	$\underset{\longleftarrow}{\uparrow < \downarrow}$	$\underset{\longrightarrow}{\downarrow > \uparrow}$	$\underset{\longrightarrow}{\downarrow > \uparrow}$
7—8	,,	,,	$\underset{\longleftarrow}{* \downarrow < \uparrow *}$
0 8—9	$\underset{\longrightarrow *}{\uparrow < \downarrow}$	,,	$\underset{\longrightarrow *}{\uparrow < \downarrow}$

TABLE V. Parchment. Procent of weight.
$X = NaCl$ $Y = Na_2CO_3$ $W = H_2O$

	t	X	Y	X	Y	X	Y	W		
1	0	0	8.435	26.580	0					
2	4½	0.245	8.402	26.250	0.0514	40.2	7.1	52.7	IV	→
3	26	1.578	8.273	24.758	0.2493	36.6	5.8	57.6	,,	,,
4	27	4.805	7.950	21.585	0.699	34.6	5.8	59.6	,,	,,
5	15	7.312	7.620	19.347	1.070	35.8	7.2	57.0	,,	,,
6	174	9.188	7.255	17.627	1.408	48.9	9.7	41.4	,,	,,
7	242	10.913	6.742	16.116	1.850	53.8	15.2	31.0	VII	←
8	317	11.986	6.262	15.106	2.285	71.0	26.9	2.1	,,	,,
9	410	12.503	5.758	14.373	2.817	18.7	0.4	80.9	VI	,,
10	509	12.922	5.332	14.002	3.320	11.3	7.0	81.7	I	→

SCHEME V.

	X	Y	W
1—8	$\underset{\longleftarrow}{\uparrow < \downarrow}$	$\underset{\longrightarrow}{\downarrow > \uparrow}$	$\underset{\longrightarrow}{\downarrow > \uparrow}$
8—9	"	"	$* \underset{\longrightarrow}{\downarrow < \uparrow} *$
0 9—10	$\underset{\longrightarrow *}{\uparrow < \downarrow}$	"	$\underset{\longrightarrow *}{\uparrow < \downarrow}$

Leiden, Lab. of inorg. Chemistry. *(To be continued).*

Mathematics. — *Ueber eine Abstandsformel in der Theorie der orthogonalen Differentialinvarianten der Kurven des R_n.* By Prof. Dr. R. WEITZENBÖCK.

(Communicated at the meeting of January 28, 1928.)

Wir leiten hier eine besonders einfache Formel her für die Abstände der oskulierenden linearen Räume einer regulären Kurve des R_n vom Koordinatenanfangspunkte.

Sind $y_i = y_i\,(t)$ $(1 = 1, 2, \dots, n)$ die Cartesischen Koordinaten des Punktes y einer regulären Kurve C des R_n, dann ist in y der oskulierende R_d festgelegt durch die Matrix

$$\left\| \begin{matrix} y_1\, y_2 \dots\dots y_n & 1 \\ y'_1\, y'_2 \dots\dots y'_n & 0 \\ \cdot\ \ \cdot\ \ \cdot\ \ \cdot\ \ \cdot\ \ \cdot & \cdot \\ \cdot\ \ \cdot\ \ \cdot\ \ \cdot\ \ \cdot\ \ \cdot & \cdot \\ y_1^{(d)} y_2^{(d)} \dots\dots y_n^{(d)} & 0 \end{matrix} \right\| \qquad y_i^{(h)} = \frac{d^h y_i}{dt^h} \quad \dots\dots \quad (1)$$

Machen wir von homogenen rechtwinkligen Koordinaten Gebrauch mit $y_{n+1} = 1$, so sind die homogenen Punktkoordinaten dieses oskulierenden R_d gegeben durch die $(d+1)$-reihigen Determinanten der Matrix (1):

$$a_{i_1 i_2 \dots i_{d+1}} = (yy'y'' \dots y^{(d)})_{i_1 \dots i_{d+1}} = (012 \dots d)_{i_1 \dots i_{d+1}} \quad \dots \quad (2)$$

wobei die Indizes i_ν jetzt die Zahlen $1, 2, \dots, n+1$ durchlaufen.

Den Abstand p_d des oskulierenden $R_d\,(a)$ von $O\,(x_i = 0)$ finden wir wie folgt. Wir projizieren O auf $R_d\,(a)$ indem wir durch O einen zu $R_d\,(a)$ total-senkrechten $R_{n-d}\,(p)$ legen und den Schnittpunkt P dieses $R_{n-d}\,(p)$ mit dem $R_d\,(a)$ ermitteln.

Die homogenen Raumkoordinaten $p'_{k_1 \dots k_d}$ des projizierenden R_{n-d} sind die d-reihigen Determinanten der Matrix, die aus (1) erhalten wird, wenn wir die erste Zeile weglassen. Bezeichnen wir dann die Reihe der $n+1$ Grössen $y_1^{(h)} : y_2^{(h)} : \dots : y_n^{(h)} : 0$ mit $y^{(h)}\,|$ oder kürzer mit $h\,|$, so haben wir:

$$p'_{k_1 \dots k_d} = (1\,|\,2\,|\, \dots d\,|)_{k_1 \dots k_d} \quad \dots\dots\dots \quad (3)$$

wobei die Indizes k_ν wieder die Zahlen $1, 2, \dots, n+1$ durchlaufen.

Die Gleichung des Punktes P in homogenen Koordinaten wird dann:

$$(u'P) = (u'p'^d\, a'^{n-d}) = 0. \quad \dots\dots\dots \quad (4)$$

Hieraus finden wir:

$$(P|P) = (P|\,p'^d\,\alpha'^{n-d}) = (n-d)!\,(\alpha|P)\,(\alpha p')^d =$$

$$= d!\,(n-d)!\,(\alpha\,|\,q'^d\,\beta'^{n-d})\,(\alpha|\,1)\,(\alpha\,|\,2)\ldots(\alpha\,|\,d)$$

Hier formen wir so um, dass alle $d+1$ Reihen $\alpha\,|$ in den Klammerfaktor hineinkommen; hierbei führen $(\beta'\,1), \ldots, (\beta'\,d)$ nach (2) auf Null, sodass bleibt:

$$(P\,|\,P) = \frac{1}{d+1}\,d!\,(n-d)!\,(\alpha\,|^{\,d+1}\,\beta'^{n-d})\,(q'1)\,(q'2)\ldots(q'd)$$

$$(P\,|\,P) = \frac{1}{d+1}\,d!\,[(n-d)!]^2\,(\beta\,|\,\alpha)^{d+1}\,(q'1)\,(q'2)\ldots(q'd) \quad . \quad . \quad (5)$$

Setzen wir jetzt

$$M_{rs} = y_1^{(r)}\,y_1^{(s)} + y_2^{(r)}\,y_2^{(s)} + \ldots + y_n^{(r)}\,y_n^{(s)}, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

dann haben wir nach (2):

$$(\beta\,|\,\alpha)^{d+1} = (d+1)!\,\sum_1^n \beta_{i_1\ldots\ldots i_{d+1}}\,\alpha_{i_1\ldots\ldots i_{d+1}} =$$

$$= (d+1)!\,\sum_1^n (012\ldots d)_{i_1\ldots\ldots i_{d+1}}\,(012\ldots d)_{i_1\ldots\ldots i_{d+1}}$$

$$(\beta\,|\,\alpha)^{d+1} = (d+1)!\,\triangle_{012\ldots d} = (d+1)!\,\begin{vmatrix} M_{00} & M_{01} & \ldots & M_{0d} \\ M_{10} & M_{11} & \ldots & M_{1d} \\ . & . & . & . \\ M_{d0} & M_{d1} & \ldots & M_{dd} \end{vmatrix} \,. \quad (7)$$

Und analog nach (3):

$$(q'\,1)\,(q'\,2)\ldots(q'\,d) = (q'\,|\,1)\,(q'\,|\,2)\ldots(q'\,|\,d) =$$

$$= \sum_1^n q'_{k_1\ldots k_d}\,(1\,|\,2\,|\ldots d\,|\,)_{k_2\ldots k_d} = \sum_1^n (1\,|\,2\,|\ldots d\,|\,)_{k_1\ldots k_d}\,(1\,|\,2\,|\ldots d\,|\,)_{k_1\ldots k_d} =$$

$$= \triangle_{12\ldots d}.$$

Also wird:

$$(P\,|\,P) = d!^2\,[(n-d)!]^2\,\triangle_{012\ldots d}\,\triangle_{12\ldots d}. \quad . \quad . \quad . \quad . \quad (8)$$

Bedeutet ferner l' die Grössenreihe $0:0:\ldots:0:1$, so ist das Quadrat p_d^2 des Abstandes $p_d = OP$ gegeben durch

$$p_d^2 = \frac{(P\,|\,P)}{(l'P)^2} = \frac{P_1^2 + P_2^2 + \ldots + P_n^2}{P_{n+1}^2}\,.$$

Nach (4) haben wir:

$$\left.\begin{aligned}(l'P) &= (l'p'^{d}\ a'^{n-d}) = (n-d)!\,(al')\,(ap')^{d} = \\ &= d!\,(n-d)!\sum_{1}^{n+1} a_{i_1\,..\,i_{d+1}}\,(l'1|2|\dots d|)_{i_1\dots i_{d+1}} = \\ &= \sum_{1}^{n+1} (l'1|2|\dots d|)_{i_1\dots i_{d+1}}\,(012\dots d)_{i_1\dots i_{d+1}} = d!\,(n-d)!\,\triangle_{12\dots d}\end{aligned}\right\}. \quad (9)$$

Somit kommt schliesslich:

$$p_d^2 = \frac{\triangle_{012\dots d}}{\triangle_{12\dots d}} \quad \dots\dots\dots \quad (10)$$

Es ist also z. B. der Abstand des Punktes y, der Tangente, der Schmiegungsebene, u.s.f. gegeben durch:

$$p_0^2 = M_{00}\ ,\quad p_1^2 = \frac{\begin{vmatrix} M_{00} & M_{01} \\ M_{10} & M_{11} \end{vmatrix}}{M_{11}}\ ,\quad p_2^2 = \frac{\begin{vmatrix} M_{00} & M_{01} & M_{02} \\ M_{10} & M_{11} & M_{12} \\ M_{20} & M_{21} & M_{22} \end{vmatrix}}{\begin{vmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{vmatrix}}\ . \quad (11)$$

Mathematics. — *Ueber Verallgemeinerungen des Satzes von* Desargues.

By Prof. Dr. R. Weitzenböck.

(Communicated at the meeting of January 28, 1928.)

Sind $a. b, c$ und α, β, γ zwei Dreiecke der projektiven Ebene mit den Seiten a', b', c' bzw. α', β', γ' und ist $f = (a\,b\,c) \neq 0$, $\varphi = (\alpha\,\beta\,\gamma) \neq 0$, dann besagt der Satz von Desargues: Gehen die drei Geraden $a\alpha$, $b\beta$ und $c\gamma$ durch einen Punkt, dann liegen die drei Punkte $a'\alpha'$, $b'\beta'$, $c'\gamma'$ auf einer Geraden und umgekehrt.

Dieser Satz ist durch J. V. Poncelet, M. Chasles, O. Hermes, F. Schur und andere auf verschiedene Arten auf zwei Tetraeder des dreidimensionalen Raumes verallgemeinert worden [1]). Die Frage nach einem analogen Satze bei drei Tetraedern wurde von W. Fr. Meyer [2]) behandelt. Wir beweisen hier, dass es bei drei beliebigen Tetraedern eine derartige Verallgemeinerung nicht gibt.

Ferners geben wir zwei einfache Verallgemeinerungen für den projektiven Raum von vier und von fünf Dimensionen und schliessen mit einigen Bemerkungen über die möglichen Verallgemeinerungen bei n Dimensionen.

§ 1.

Der Satz von Desargues kann analytisch am einfachsten nach E. Hunyadi [3]) wie folgt bewiesen worden. Dass $a\alpha$, $b\beta$, $c\gamma$ durch einen Punkt gehen, wird ausgedrückt durch

$$K = ((a\alpha)\ (b\beta)\ (c\gamma)) = 0 \quad \ldots\ldots\ldots \quad (1)$$

Dies ist wegen $((a\alpha)\,u'v') = (au')\,(\alpha v') - (av')\,(\alpha u')$ gleichbedeutend mit:

$$K = \begin{vmatrix} (ab\beta) & (\alpha b\beta) \\ (ac\gamma) & (\alpha c\gamma) \end{vmatrix} = \begin{vmatrix} (c'\beta) & -(\gamma' b) \\ -(b'\beta) & (\beta' c) \end{vmatrix} = 0. \quad \ldots\ldots \quad (2)$$

Die zu K duale Invariante ist $K' = ((a'\alpha')\ (b'\beta')\ (c'\gamma'))$.

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[1]) F. Schur, Math. Ann. **19** (1882), p. 429—432. Ausführliche Litteratur hierüber findet man bei A. Baruch, Rend. di Palermo **44** (1920), p. 261—300. Hiezu noch: E. Study, Marburger Ber. (1900), p. 78; G. Kohn, Jahresber. d. Deutschen Math. Ver. **22** (1913), p. 343 und Wiener Ber. **127** (1918) p. 2073 und 2088. Vgl. auch den Art. III C 8 von K. Zindler der Math. Encykl., Nr. 2. (1922).

[2]) W. Fr. Meyer, Archiv der Math. u. Phys. **1** (1901), p. 372.

[3]) E. Hunyadi, Crelle **139** (1879), p. 79.

Setzen wir hier $a'_l = (bc)_{ik}$ und $\alpha'_l = (\beta\gamma)_{ik}$, so entsteht: .

$$K = (abc)\,.\,(\alpha\beta\gamma)\,.\begin{vmatrix} (\beta' c) & -(b'\gamma) \\ -(\gamma' b) & (c'\beta) \end{vmatrix} = f\,.\,\varphi\,.\,K. \quad . \quad . \quad . \quad (3)$$

Also verschwinden K und K' gleichzeitig.

Dieser Beweis setzt auch den Inhalt des Satzes in deutliches Licht: Eine Invariante der drei Verbindingslinien der Eckpunkte der beiden Dreiecke verschwindet gleichzeitig mit der dualen, aus den Schnittpunkten der Seiten aufgebauten Invariante. Diese Fassung führt unmittelbar auf die Frage nach analogen Sätzen bei mehr als zwei Dimensionen.

Im dreidimensionalen Raume geben zunächst zwei Tetraeder $abcd$ und $\alpha\beta\gamma\delta$ mit $f = (abcd) \neq 0$ und $\varphi = (\alpha\beta\gamma\delta) \neq 0$ die vier Verbindungslinien $a\alpha$, $b\beta$, $c\gamma$, $d\delta$. Hier haben bereits 2 Geraden, z. B. $a\alpha$ und $b\beta$ eine relatieve Invariante:

$$A_{12} = \Sigma\,(a\alpha)_{12}\,(b\beta)_{34} = (a\alpha b\beta). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Sind a', b', c', d' und α', β', γ', δ' die Seiten der Tetraeder, ist also

$$a'_1 = +(bcd)_{234},\ a'_2 = -(bcd)_{134},\ a'_3 = +(bcd)_{124},\ a'_4 = -(bcd)_{123}$$
$$b'_1 = -(acd)_{234},\ b'_2 = +(acd)_{134},\ b'_3 = -(acd)_{124},\ b'_4 = +(acd)_{123}$$

etc., so ist mit A_{12} dual: $A'_{12} = (a'\alpha' b'\beta')$. Nun ist aber:

$$(a'b')_{ik} = f\,.\,(cd)_{rs} \text{ und } (\alpha'\beta')_{rs} = \varphi\,.\,(\gamma\delta)_{ik};$$

also wird:

$$A'_{12} = f\,.\,\varphi\,.\,A_{34} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

was unmittelbar geometrisch zu deuten ist.

Aus (5) leitet man weiters ab, dass auch für die Invariante

$$K = \begin{vmatrix} O & A_{12} & A_{13} & A_{14} \\ A_{21} & O & A_{23} & A_{24} \\ A_{31} & A_{32} & O & A_{34} \\ A_{41} & A_{42} & A_{43} & O \end{vmatrix} \quad (A_{ik} = A_{ki}) \quad . \quad . \quad . \quad . \quad (6)$$

eine analoge Gleichung gilt: $K' = f^4\,.\,\varphi^4\,.\,K$, woraus diejenigen verallgemeinerungen des DESARGUES'schen Satzes gewonnen werden, die sich um die hyperboloidische Lage der zwei Tetraeder gruppieren.

Bei drei Tetraedern $ABCD$, $abcd$ und $\alpha\beta\gamma\delta$ hat schon W. FR. MEYER darauf hingewiesen, dass die naheliegende Verallgemeinerung: Gehen die vier Ebenen $Aa\alpha, \ldots$ durch einen Punkt, dann liegen die vier Punkte $A'a'\alpha', \ldots$ in einer Ebene, nicht allgemein richtig ist.

Beweis. Dass $Aa\alpha$, $Bb\beta, \ldots$ durch einen Punkt gehen, wird ausgedrückt durch das Verschwinden der Invariante

$$K = (\,(Aa\alpha)\,(Bb\beta)\,(Cc\gamma)\,(Dd\delta)\,) = \begin{vmatrix} (ABb\beta) & (abB\beta) & (\alpha\beta bB) \\ (ACc\gamma) & (acC\gamma) & (\alpha\gamma cC) \\ (ADd\delta) & (adD\delta) & (\alpha\delta dD) \end{vmatrix} \quad . \quad . \quad (7)$$

Dual hiezu wird die Bedingung, dass die vier Punkte $A'a'\alpha', \ldots$ in einer Ebene liegen, gegeben durch das Verschwinden der Invariante

$$K' = ((A'a'\alpha')(B'b'\beta')(C'c'\gamma')(D'd'\delta')) = \begin{vmatrix} (A'B'b'\beta') & (a'b'B'\beta') & (\alpha'\beta'b'B') \\ (A'C'c'\gamma') & (a'c'C'\gamma') & (\alpha'\gamma'c'C') \\ (A'D'd'\delta') & (a'd'D'\delta') & (\alpha'\delta'd'D') \end{vmatrix}.$$

Drücken wir hier A', a' und α' durch BCD, bcd und $\beta\gamma\delta$ aus, so entsteht, wenn wir die Abkürzung

$$(u'v')_{(xy)} = (u'x)(v'y) - (u'y)(v'x)$$

verwenden:

$$K' = F \,.\, f \,.\, \varphi \,.\, \begin{vmatrix} (b'\beta')_{(CD)} & (\beta'B')_{(cd)} & (B'b')_{(\gamma\delta)} \\ (c'\gamma')_{(DB)} & (\gamma'C')_{(db)} & (C'c')_{(\delta\beta)} \\ (d'\delta')_{(BC)} & (\delta'D')_{(bc)} & (D'd')_{(\beta\gamma)} \end{vmatrix}.$$

Wäre nun obiger Satz richtig, so müssten K' und K gleichzeitig verschwinden. Dass dies nicht der Fall ist, lässt sich durch folgendes Beispiel zeigen. Wir wählen $ABCD$ als Koordinatensimplex. Dann wird $F = 1$ und nach (7) haben wir:

$$K = \begin{vmatrix} b_3\beta_4 - b_4\beta_3 & -(ab\beta)_{134} & (\alpha\beta b)_{134} \\ c_4\gamma_2 - c_2\gamma_4 & (ac\gamma)_{124} & -(\alpha\gamma c)_{124} \\ d_2\delta_3 - d_3\delta_2 & -(ad\delta)_{123} & (\alpha\delta d)_{123} \end{vmatrix}.$$

Dagegen kommt statt K':

$$K' = f \,.\, \varphi \,.\, \begin{vmatrix} -(acd)_{124}\beta'_4 - (acd)_{123}\beta'_3 & (\beta'c)d_2 - (\beta'd)c_2 & (a\delta cd)\gamma_2 - (a\gamma cd)\delta_2 \\ -(abd)_{123}\gamma'_2 + (abd)_{134}\gamma'_4 & (\gamma'd)b_3 - (\gamma'b)d_3 & (ab\beta d)\delta_3 - (ab\delta d)\beta_3 \\ +(abc)_{134}\delta'_3 + (abc)_{124}\delta'_2 & (\delta'b)c_4 - (\delta'c)b_4 & (abc\gamma)\beta_4 - (abc\beta)\gamma_4 \end{vmatrix}.$$

Nun spezialisieren wir wie folgt:

$$f = \begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & 0 & b_4 \\ c_1 & 0 & c_3 & c_4 \\ d_1 & d_2 & 0 & d_4 \end{vmatrix} \neq 0, \quad \varphi = \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \beta_1 & \beta_2 & 0 & 0 \\ \gamma_1 & 0 & \gamma_3 & \gamma_4 \\ 0 & 0 & 0 & \delta_4 \end{vmatrix} \neq 0.$$

Mit diesen Werten wird $K = 0$, dagegen

$$K' = -f \,.\, \varphi\,(abc\beta) \,.\, \gamma_4 \,.\, \alpha_2\delta_4\,(\gamma c)_{13} \,.\, a_3\,(bd)_{12} \,.\, \delta_4\alpha_3\beta_1 \neq 0.$$

§ 3.

Im projektiven R_4 gehen wir aus von zwei Simplexen $abcde$ und $\alpha\beta\gamma\delta\varepsilon$. Die 5 Geraden $a\alpha, \ldots, e\varepsilon$ haben unter anderem die folgende Invariante:

$$K = \Sigma\,(c\gamma)_{ik}\,(a\alpha b\beta)_i\,(d\delta e\varepsilon)_k = (a\alpha b\beta c)(d\delta e\varepsilon\gamma) - (a\alpha b\beta\gamma)(d\delta e\varepsilon c) \quad . \qquad (8)$$

Ihr Verschwinden sagt aus, dass die Schnittebene der beiden R_3 $a\alpha b\beta$ und $d\delta e\varepsilon$ von der Geraden $c\gamma$ getroffen wird.

Dual zu K haben wir:

$$K' = (a'\alpha'b'\beta'c')(d'\delta'e'\varepsilon'\gamma') - (a'\alpha'b'\beta'\gamma')(d'\delta'e'\varepsilon'c');$$

Hier ist wegen

$$(a'b'c')_{ikl} = f^2 \, . \, (de)_{rs}, \quad (\alpha'\beta')_{ik} = \varphi \, . \, (\gamma\delta\varepsilon)_{rst}$$

der erste Klammerfaktor von K' auch gleich $f^2 . \varphi . (d\delta e\varepsilon\gamma)$ und analog die übrigen Faktoren. Dies gibt

$$K' = f^3 . \varphi^3 . K, \quad \ldots\ldots\ldots \quad (9)$$

was wieder leicht geometrisch zu deuten ist.

Nimmt man in (8) $a\alpha$ statt $d\delta$, geht also aus von der Invariante

$$J = \Sigma\, (c\gamma)_{ik}\, (a\alpha b\beta)_i\, (a\alpha e\varepsilon)_k = (a\alpha b\gamma c)\, (a\alpha e\varepsilon\gamma) - (a\alpha b\beta\gamma)\, (a\alpha e\varepsilon c), \quad . \quad (10)$$

dann erhält man für die zu J duale Invariante J':

$$J' = \Sigma\, (c'\gamma')_{ik}\, (a'\alpha'b'\beta')_i\, (a'\alpha'e'\varepsilon')_k =$$

$$= f^3 . \varphi^3 . \Sigma\, (c\gamma)_{ik} (b\beta d\delta)_i\, (d\delta e\varepsilon)_k = f^3 . \varphi^3 . J_1, \text{ wo}$$

$$J_1 = (b\beta d\delta c)\, (d\delta e\varepsilon\gamma) - (b\beta d\delta\gamma)\, (d\delta e\varepsilon c) \quad \ldots\ldots\ldots \quad (11)$$

eine von J verschiedene Invariante der fünf Geraden $a\alpha, \ldots, e\varepsilon$ ist. Es führt also J' auf $J_1 \neq J$. Wir hatten übrigens schon bei (5) § 2 ein derartiges Ergebnis.

Ein weiteres Beispiel für dieses Verhalten von J haben wir bei $n=6$, also im projektiven Raum R_5 von 5 Dimensionen. Hier haben bereits drei Verbindungslinien $a\alpha$, $b\beta$ und $c\gamma$ eine Invariante

$$A_{123} = (a\alpha b\beta c\gamma). \quad \ldots\ldots\ldots \quad (12)$$

Für $A'_{123} = (a'\alpha'b'\beta'c'\gamma')$ finden wir:

$$A'_{123} = f^2 . \varphi^2 . (e\varepsilon f\eta g\zeta) = f^2 . \varphi^2 . A_{456}. \quad \ldots\ldots \quad (13)$$

Hier ist $A'_{123} = 0$ die Bedingung dafür, dass die drei Geraden $a\alpha$, $b\beta$ und $c\gamma$ einem linearen R_4 angehören.

§ 4.

Im allgemeinen Falle gehen wir bei $n-1$ Dimensionen aus von zwei Simplexen $abc \ldots lm$ und $\alpha\beta\gamma \ldots \lambda\mu$ und haben n Verbindungslinien $a\alpha, \ldots, m\mu$.

Sei $K(h_1, h_2, \ldots, h_n)$ eine ganze rationale projektive Invariante dieser Geraden, vom Grade h_1 in den $(a\alpha)_{ik}$, vom Grade h_2 in den $(b\beta)_{ik}$, . K ist dann eine Summe von Termen, jeder Term ist ein Produkt von $\frac{2s}{n}$ Klammerfaktoren $(a\alpha b\beta c\gamma d\delta \ldots \varepsilon\eta \ldots)$, wobei $s = h_1 + h_2 + \ldots + h_n$.

Gehen wir zur dualen Invariante K' über, so ergibt das Zurückgehen zu den Reihen $a, b, c, \ldots \alpha, \beta, \gamma, \ldots$ die Gleichung

$$K'(h_1, h_2, \ldots, h_n) = (f . \varphi)^{\frac{s}{n}\cdot(n-2)} . K_1 \left(\frac{2s}{n} - h_1, \frac{2s}{n} - h_2, \ldots, \frac{2s}{n} - h_n\right). \quad (14)$$

wo die positiven Zahlen $\frac{2s}{n} - h_1 \ldots$ die Graden von K_1 in $(a\alpha)_{ik}, \ldots$ angeben.

(14) *stellt die umfassendste Verallgemeinerung des* DESARGUEUS'*schen Satzes auf zwei Simplexe des* R_{n-1} *dar.*

Es ergibt sich noch die Frage: wann ist K_1 mit K identisch?

Eine notwendige Bedingung ist leicht anzugeben. Aus (14) entnimmt man nämlich für diesen Fall:

$$\frac{2s}{n} - h_i = h_i, \text{ also } h_1 = h_2 = \ldots = h_n = \frac{s}{n}.$$

Es muss also K von gleichem Grade in allen Geraden $a\alpha$, $b\beta, \ldots$ sein. Ob dies für $K_1 = K$ auch hinreicht, dürfte wohl erst dann zu entscheiden sein, wenn man über die Struktur von K Näheres weis.

Mathematics. — *Ueber eine Configuration von zehn Geraden im projekliven* R_3. By Prof. Dr. R. WEITZENBÖCK.

(Communicated at the meeting of January 28, 1828.)

Zu vier Geraden des Raumes gibt es zwei, eine oder unendlich viele Transversalen, d.h. Geraden, die alle vier schneiden. Wir betrachten im Folgenden den Fall, dass vier Geraden genau eine Transversale besitzen oder, wie wir sagen wollen, ein "einfach-singuläres" Quadrupel bilden.

Aus fünf Geraden K_1, K_2, K_4, K_4 und K_5 lassen sich fünf Quadrupel Q_i bilden. *Wir wollen nachweisen, dass es Figuren von fünf Geraden gibt, bei denen jedes Quadrupel* Q_i *einfach-singulär ist.* Man erhält so ein zweites Quintupel $L_1, L_2, \ldots, L_5$ von Geraden, wobei L_i die Transversale von Q_i ist. Die Beziehung zwischen den Quintupeln K und L ist reziprok und alle 10 Geraden, von denen höchstens nur 6 reell sein können, gehören einem linearen Komplexe an.

§ 1.

Sind a_{ik}, b_{ik}, c_{ik}, d_{ik} und e_{ik} die homogenen Punktkoordinaten von fünf Geraden, ist also z.B.

$$K_1 = \Sigma\, \pi'_{ik} a_{ik} = \Sigma\, \pi_{rs} a_{ik} = \tfrac{1}{2} (\pi' a)^2 = \tfrac{1}{2} (\pi a')^2 = 0$$

die Gleichung der Geraden a_{ik}, so haben wir 10 relative Invarianten $A_{ik} = A_{ki}$, wo z.B. $A_{12} = \Sigma\, a'_{ik} b_{ik} = \Sigma\, a_{rs} b_{ik} = a_{12} b_{34} + a_{13} b_{42} + a_{14} b_{23} + a_{34} b_{12} + a_{42} b_{13} + a_{23} b_{14}$ ist.

Die Fläche zweiter Ordnung F_{123}, die durch K_1, K_2 und K_3 bestimmt wird, hat in Linienkoordinaten die Gleichung:

$$\Pi_{123} = \begin{vmatrix} 0 & A_{12} & A_{13} & K_1 \\ A_{21} & 0 & A_{23} & K_2 \\ A_{31} & A_{32} & 0 & K_3 \\ K_1 & K_2 & K_3 & 0 \end{vmatrix} = 0 \quad . \; . \; . \; . \; . \; . \; . \quad (1)$$

Hieraus findet man als Bedingung, dass K_4 diese F_{123} berührt:

$$A^{55} = \begin{vmatrix} 0 & A_{12} & A_{13} & A_{14} \\ A_{21} & 0 & A_{23} & A_{24} \\ A_{31} & A_{32} & 0 & A_{34} \\ A_{41} & A_{42} & A_{43} & 0 \end{vmatrix} = 0 \quad . \; . \; . \; . \; . \; . \; . \quad (2)$$

Ferner wird das Produkt der beiden Transversalen zu K_1, K_2, K_3 und K_4 gegeben durch:

$$\Pi_{1234} = \begin{vmatrix} 0 & A_{12} & A_{13} & A_{14} & K_1 \\ A_{21} & 0 & A_{23} & A_{24} & K_2 \\ A_{31} & A_{32} & 0 & A_{34} & K_3 \\ A_{41} & A_{42} & A_{43} & 0 & K_4 \\ K_1 & K_2 & K_3 & K_4 & 0 \end{vmatrix} = 0. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Das Quadrupel $Q_5 = (K_1, K_2, K_3, K_4)$ ist einfach-singulär, wenn $A^{55} = 0$, $\Pi_{1234} \equiv/\equiv 0$. Bei $\Pi_{1234} \equiv 0$ ist Q_5 hyperboloidisch.

Ist Q_5 einfach-singulär, so wird Π_{1234} ein Quadrat und die Transversale L_5 kann dann dargestellt werden durch

$$\begin{vmatrix} 0 & A_{12} & A_{13} & A_{14} & K_1 \\ A_{21} & 0 & A_{23} & A_{24} & K_2 \\ A_{31} & A_{32} & 0 & A_{34} & K_3 \\ A_{41} & A_{42} & A_{43} & 0 & K_4 \\ A_{51} & A_{52} & A_{53} & A_{54} & 0 \end{vmatrix} = 0.$$

Nennen wir $A \neq 0$ die fünfreihige Determinante $|A_{ik}|$ und A^{ik} die Minoren von A_{ik}, so lautet diese Gleichung: $\Sigma\, K_i\, A^{i5} = K_i\, A^{i5} = 0$. Allgemein haben wir also, wenn alle fünf Quadrupel Q_i einfach-singulär sein sollen:

$$A^{ii} = 0 \quad , \quad L_i = \sum_{\lambda=1}^{\lambda=5} K_\lambda\, A^{\lambda i} = K_\lambda\, A^{\lambda i} = 0 \quad . \quad . \quad . \quad . \quad (4)$$

Hieraus folgt wegen $A \neq 0$ leicht die Reziprozität der Beziehung zwischen den beiden Quintupeln K und L. Die simultane Invariante $(L_i \cdot L_k)$ wird nämlich nach (4): $A_{\lambda\mu}\, A^{\lambda i}\, A^{\mu k} = A \,.\, \delta^i_\mu\, A^{\mu k} = A \,.\, A^{ik}$. Nennen wir also L'_i die Gerade, die aus den L so entsteht wie L_i aus den K, so wird:

$$L'_i = L_\lambda \,.\, A^4 \,.\, A^3 \,.\, A_{\lambda i} = A^7 \,.\, K_\mu\, A^{\mu\lambda}\, A_{\lambda i} = A^8 \,.\, K_i \,.$$

Bei $A \neq 0$ ist also L'_i mit K_i identisch, während $A = 0$ den Ausnahmefall gibt, wo alle K eine einzige Transversale L besitzen.

§ 2.

Wir konstruieren ein Quintupel K mit einfach-singulären Quadrupeln Q_i wie folgt. Auf F_{123} nehmen wir zwei Punkte P_{45} und P_{54} so an, dass ihre Verbindungslinie keine Erzeugende von F_{123} ist. In diesen Punkten legen wir Tangenten K_4 bzw. K_5 an F_{123}. Dann sind die beiden Quadrupel Q_4 und Q_5 bereits einfach-singulär: $A^{55} = 0$, $A^{44} = 0$. Hierauf versuchen wir die restlichen drei Gleichungen $A^{ii} = 0$ zu befriedigen.

Wir legen das Koordinatentetraeder 1234 so, dass $13 = K_1$ und $24 = K_2$ wird, dass P_{45} auf 12 und P_{54} auf 34 zu liegen kommt. (Den trivialen

Fall, dass alle fünf Geraden K eine einzige Transversale besitzen, lassen wir beiseite).

F_{123} hat dann die Gleichung $x_1 x_4 - x_2 x_3 = 0$ und die Erzeugendenschaar, welcher K_1, K_2 und K_3 angehören, wird gegeben durch:

$$x_2 - \lambda x_1 = 0 \quad , \quad x_4 - \lambda x_3 = 0.$$

$\lambda = 0$ gibt K_1, $\lambda = \infty$ gibt K_2 und K_3 sei festgelegt durch $\lambda = r$. K_3 schneidet dann 12 im Punkte $P_{35} = 1 : r : 0 : 0$ und 34 im Punkte $P_{34} = 0 : 0 : 1 : r$. P_{45} und P_{54} wollen wir auf 12 bzw. auf 34 festlegen durch zwei von r verschiedene Zahlen p und q, sodass:

$$P_{45} = 1 : p : 0 : 0 \quad \text{und} \quad P_{54} = 0 : 0 : 1 : q.$$

K_4 muss F_{123} berühren, muss also in der durch 12 und die Erzeugende $\lambda = p$ bestimmten Ebene liegen; man kann also K_4 festlegen durch:

$$K_4 = \begin{Vmatrix} 1 & p & 0 & 0 \\ 1 & p' & \varrho & \varrho p \end{Vmatrix} \quad \text{mit} \quad p' \neq p \quad , \quad \varrho \neq 0.$$

Analog bei K_5:

$$K_5 = \begin{Vmatrix} 0 & 0 & 1 & q \\ 1 & q & \sigma & \sigma q' \end{Vmatrix} \quad \text{mit} \quad q' \neq q \quad , \quad \sigma \neq 0$$

Auf diese Weise erhält man als Koordinaten $\pi_{12} : \pi_{13} : \pi_{14} : \pi_{34} : \pi_{42} : \pi_{23}$ der Geraden K_i:

$$\left.\begin{array}{llllllllllll} K_1 \ldots & 0 & : & 1 & : & 0 & : & 0 & : & 0 & : & 0 \\ K_2 \ldots & 0 & : & 0 & : & 0 & : & 0 & : & 1 & : & 0 \\ K_3 \ldots & 0 & : & 1 & : & r & : & 0 & : & -r^2 & : & r \\ K_4 \ldots & p' - p & : & \varrho & : & \varrho q & : & 0 & : & -p^2\varrho & : & p\varrho \\ K_5 \ldots & 0 & : & -1 & : & -q & : & \sigma(q'-q) & : & q^2 & : & -q \end{array}\right\} \quad . \quad . \quad (5)$$

Hieraus findet man:

$$|A_{ik}| = \begin{vmatrix} 0 & 1 & -r^2 & -p^2\varrho & q^2 \\ & 0 & 1 & \varrho & -1 \\ & & 0 & -\varrho(p-r)^2 & (q-r)^2 \\ & & & 0 & \sigma(p'-p)(q'-q) + \varrho(p-q)^2 \\ & & & & 0 \end{vmatrix} , \quad . \quad . \quad (6)$$

$$A^{11} = \sigma(p'-p)(q'-q)\left[\sigma(p'-p)(q'-q) - 4\varrho(p-r)(q-r)\right]$$
$$A^{22} = r^2\sigma(p'-p)(q'-q)\left[r^2\sigma(p'-p)(q'-q) - 4pq\varrho(p-r)(q-r)\right]$$
$$A^{33} = \sigma(p'-p)(q'-q)\left[\sigma(p'-p)(q'-q) - 4pq\varrho\right].$$

Setzen wir $A^{ii} = 0$, so folgt leicht: $r^2 = pq$, $r = p + q$, $p^3 - q^q = 0$. Wegen $p \neq q$ gibt dies, wenn ε eine primitieve dritte Einheitswurzel bedeutet $(\varepsilon^2 + \varepsilon + 1 = 0)$:

$$p = -r\varepsilon \qquad q = -r\varepsilon^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

wozu dann noch für p', q', ϱ und σ die Beziehung kommt:

$$\sigma (p' + \varepsilon r)(q' + \varepsilon^2 r) = 4 \varrho r^2 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

Nach (5) liegt K_4 fest, wenn $\frac{p' - p}{\varrho} = \frac{p' + \varepsilon r}{\varrho}$ gegeben ist und K_5 ist durch $\sigma (q' - q) = \sigma (q' + \varepsilon^2 r)$ fixiert. Nun ist aber nach (8): $\frac{p' + \varepsilon r}{\varrho} = \frac{4r^2}{\sigma (q' + \varepsilon^2 r)}$; setzen wir also $s = \sigma (q' + \varepsilon^2 r)$, so werden K_4 und K_5 durch s festgelegt. Statt (5) kommt dann:

$$\left.\begin{array}{lcccccc}
K_1 \ldots & 0 & : \; 1 & : \; 0 & : \; 0 & : \; 0 & : \; 0 \\
K_2 \ldots & 0 & : \; 0 & : \; 0 & : \; 0 & : \; 1 & : \; 0 \\
K_3 \ldots & 0 & : \; 1 & : \; r & : \; 0 & : - r^2 & : \; r \\
K_4 \ldots & \frac{4r^2}{s} & : \; 1 & : - \varepsilon r & : \; 0 & : - \varepsilon^2 r^2 & : - \varepsilon r \\
K_5 \ldots & 0 & : - 1 & : \; \varepsilon^2 r & : \; s & : \; \varepsilon r^2 & : \; \varepsilon^2 r
\end{array}\right\} \quad . \; . \quad (9)$$

und die A_{ik} werden jetzt:

$$|A_{ik}| = \begin{vmatrix}
0 & 1 & -r^2 & -\varepsilon^2 r^2 & \varepsilon r^2 \\
 & 0 & 1 & 1 & -1 \\
 & & 0 & -\varepsilon r^2 & \varepsilon^2 r^2 \\
 & & & 0 & r^2 \\
 & & & & 0
\end{vmatrix}, \quad . \; . \; . \; . \; . \; . \; . \quad (10)$$

Mit diesen Werten finden wir:

$$|A^{ik}| = \begin{vmatrix}
0 & 8r^6 & -8r^4 & -8\varepsilon r^4 & 8\varepsilon^2 r^4 \\
 & 0 & 8r^6 & 8r^6 & -8r^6 \\
 & & 0 & -8\varepsilon^2 r^4 & 8\varepsilon r^4 \\
 & & & 0 & 8r^4 \\
 & & & & 0
\end{vmatrix}, A = 32 r^6 \quad . \; . \; . \quad (11)$$

und also nach (4):

$$\left.\begin{array}{l}
L_1 = 8r^4 . [\qquad\qquad\qquad r^2 . K_2 \; - 1 . K_3 \; - \varepsilon . K_4 \; + \varepsilon^2 . K_5] \\
L_2 = 8r^4 . [\quad r^2 . K_1 \qquad\qquad\quad + r^2 . K_3 \; + r^2 . K_4 \; - r^2 . K_5] \\
L_3 = 8r^4 . [- 1 . K_1 \; + r^2 . K_2 \qquad\qquad\quad - \varepsilon^2 . K_4 \; + \varepsilon . K_5] \\
L_4 = 8r^4 . [- \varepsilon . K_1 \; + r^2 . K_2 \; - \varepsilon^2 . K_3 \qquad\qquad\quad + 1 . K_5] \\
L_5 = 8r^4 . [\quad \varepsilon^2 . K_1 \; - r^2 . K_2 \; + \varepsilon . K_3 \; + 1 . K_4 \qquad\qquad\quad]
\end{array}\right\} \quad (12)$$

Um allgemeine Gleichungen zu erhalten, kann man r^2 und ε durch GRASSMANN'sche Doppelverhältnisse ausdrücken. Nach (10) haben wir nämlich:

$$\frac{A_{12} A_{34}}{A_{23} A_{14}} = - \frac{\varepsilon}{r^2} \; , \quad \frac{A_{14} A_{23}}{A_{12} A_{34}} = \varepsilon \quad . \; . \; . \; . \; . \; . \quad (13)$$

Hieraus sind ε und r^2 durch absolute Invarianten auszudrücken und können in (12) eingesetzt werden.

Nach (12) und (9) ergeben sich für die homogenen Punktkoordinaten der Geraden L_i die Werte:

$$\left.\begin{array}{llllllllllll} L_1 \ldots & -\dfrac{4\,\varepsilon r^2}{s} & : & 0 & : & -2r & : & \varepsilon^2 s & : & 4r^2 & : & -2r \\ L_2 \ldots & \dfrac{4\,r^2}{s} & : & 4 & : & 2r & : & -s & : & 0 & : & 2r \\ L_3 \ldots & -\dfrac{4\,\varepsilon^2 r^2}{s} & : & 0 & : & 2r & : & \varepsilon s & : & 0 & : & 2r \\ L_4 \ldots & 0 & : & 0 & : & 0 & : & 1 & : & 0 & : & 0 \\ L_5 \ldots & 1 & : & 0 & : & 0 & : & 0 & : & 0 & : & 0 \end{array}\right\} . \quad (14)$$

Die zehn Geraden K und L schneiden sich in 20 Punkten $P_{ik} = (K_i L_k)$, $i \neq k$. Wir erhalten eine in sich duale Configuration mit der Signatur

$$(20_2^7\ ,\ 10_4^4\ ,\ 20_7^2).$$

Schliesslich stellen wir noch die Koordinaten der 20 Punkte P_{ik} (bezogen auf das Grundtetraeder $1 = P_{15}$, $2 = P_{25}$, $3 = P_{14}$, $4 = P_{24}$) in nachfolgender Tabelle zusammen:

	K_1	K_2	K_3	K_4	K_5
L_1	★	$0 : 2\varepsilon r : 0 : s$	$2\varepsilon r : 2\varepsilon r^2 : -s : -sr$	$2r : 2r^2(1-\varepsilon^2) : -\varepsilon^2 s : sr$	$2\varepsilon r : -2r^2 : -s \quad sr(\varepsilon-1)$
L_2	$2r : 0 : -s : 0$	★	$2r : 2r^2 : s : sr$	$2r(1-\varepsilon) : 2r^2 : -\varepsilon^2 s : sr$	$-2\varepsilon r : 2r^2 : s(1-\varepsilon^2) : sr$
L_3	$2\varepsilon^2 r : 0 : s : 0$	$0 : 2\varepsilon^2 r : 0 : -s$	★	$2\varepsilon^2 r : 2r^2 : s : -\varepsilon sr$	$2\varepsilon^2 r : -2\varepsilon r^2 : s : \varepsilon^2 sr$
L_4	$0 : 0 : 1 : 0$	$0 : 0 : 0 : 1$	$0 : 0 : 1 : r$	★	$0 : 0 : 1 . -\varepsilon^2 r$
L_5	$1 : 0 : 0 : 0$	$0 : 1 : 0 : 0$	$1 : r : 0 : 0$	$1 : -\varepsilon r : 0 : 0$	★

(15)

Auf eine ausführliche Untersuchung der sehr zahlreichen Eigenschaften der in obigem nachgewiesenen Configuration will ich bei späterer Gelegenheit eingehen. Hier sei nur noch Folgendes angeführt. Nach (10) haben wir:

$$U = A_{12}\,A_{34} = -\,\varepsilon r^2, \quad V = A_{13}\,A_{42} = -\,r^2, \quad W = A_{14}\,A_{23} = -\,\varepsilon^2 r^2. \qquad (16)$$

woraus $U + V + W = 0$ und $VW + WU + UV = 0$ folgen. Es gehört also z.B. K_4 dem durch K_1, K_2 und K_3 bestimmten VOSSE'schen Komplexe an und K_1, K_2, K_3 und K_4 bilden ein sogenanntes KLUYVER'sches Quadrupel [1]).

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[1]) Hiezu: H. MOHRMANN, Mathem. Zeitschr. 2 (1918) p. 27—51 und 5 (1919) p. 268—283.